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Modal identification of thin-walled steel studs under non-uniform temperature

Jean C. Batista Abreu¹, Zhanjie Li²

Abstract

This paper investigates complex instability phenomena of cold-formed steel members subjected to fire, using a modal identification approach based on the constrained finite strip method. During fire, elevated temperatures produce degradation of mechanical properties and thermal deformations on steel members, altering their expected performance at ambient conditions. The non-uniform temperature distribution through the cross-section of thin-walled members changes over time of fire exposure, potentially impacting the interaction of buckling modes. For instance, members normally controlled by symmetric local buckling at ambient temperature may develop a significant participation of non-symmetric distortional buckling, and a more pronounced interaction of modes at elevated temperatures. A set of thin-walled cold-formed steel members is investigated under thermal gradient. Modal identification based on the deformation field obtained through shell finite element nonlinear analysis is used to quantitatively assess the evolution of modal participations in thin-walled members as the temperature, and identify how the failure modes are coupled. In addition, the results can potential reveal the impact of thermal gradients on the load-carrying capacity of structural members under fire.

1. Introduction

Due to high cross-sectional slenderness, the strength of thin-walled cold-formed steel members is often governed by its buckling behavior. The instabilities of cold-formed steel member can be commonly categorized as: local (local-plate), distortional, and global (Euler) buckling. The design specifications, such as AISI-S100 (AISI 2012), require appropriate separation and identification of the buckling modes because of the different post-buckling strength and interactions between the modes.

Many advanced methods have been developed over the year to seek this separation and identification capability, most notably, the constrained Finite Strip Method (cFSM) (Ádány & Schafer 2006; Ádány & Schafer 2006; Li et al. 2014; Li & Schafer 2010; Li & Schafer 2009; Li & Schafer 2013) and Generalized Beam Theory (GBT) (Abambres et al. 2014a; Abambres et al. 2014b; SILVESTRE & CAMOTIM 2003a; SILVESTRE & CAMOTIM 2003b). In particular,

¹ Visiting Assistant Professor, Bucknell University, <jean.batistaabreu@bucknell.edu>

² Assistant Professor, SUNY Polytechnic Institute, <zhanjie.li@sunyit.edu>

the newly developed cFSM is able to decompose buckling modes and categorize arbitrary buckling modes (or general displacements) into the fundamental deformation classes of global (G), distortional (D), local (L), shear (S), and transverse extension (T). Moreover, the base functions (or vectors) in cFSM have been employed to quantitatively identify buckling modes of shell finite element models for not only the elastic buckling analysis but also the nonlinear collapse analysis (Li et al. 2014), for which the buckling mode or the failure mode usually require a laborious and completely subjective identification procedure employing visual investigation. Specifically, the generalized base vectors in Li et al. (2011; 2013) provide a means to potentially handle arbitrary boundary conditions and loading.

Hence, the modal identification application as part of the cFSM's capability has the potential to aid the identification of failure modes for members under elevated temperatures during fire as discussed in Li et al. (2014). In this paper, the complex instability phenomena of cold-formed steel members subjected to fire are simulated using shell finite element models and their behaviors are investigated using the modal identification approach to study the potential interaction of buckling modes for cold-formed steel members under fire.

2. CFS Members Under Thermal Gradients

During fire, elevated temperatures impact the behavior of thin-walled CFS members. First, as the temperatures increase, CFS members experience thermal expansion and, therefore, their geometry and/or the distribution of stresses in the material change. Simultaneously, the strength and stiffness of the members are altered due to the degradation of mechanical properties.

In a building, fire compartments are constructed to contain smoke and flames. Partition and loadbearing walls and floors are essential components of the passive fire protection system. When fire develops in one room, there will be one surface of the wall directly exposed to fire, while the other "unexposed" side experiences a temperature increase mostly due to heat conduction through the wall and convection in the wall cavity. Therefore, the directly exposed side of the wall may develop higher temperatures compared to the unexposed side. Consequently, studs in the wall experience thermal gradients through their cross-section (from one flange to the other), as well as thermal gradients along their length due to the hot gases rising. The flange with higher temperature will then have less strength (for instance) than the other flange. Consequently, the center of resistance is shifted away from the flange with higher temperature, inducing the development of bending moments even in members designed to only carry axial load. Additionally, non-uniform thermal strains develop and, depending on the support conditions, CFS studs tend to bow towards the fire, perhaps aggravating the situation since the closer the studs are to the fire, the higher the temperatures will be.

2.1 Thermal gradient due to fire

Data displaying actual thermal gradients in steel studs is limited; however, reports show that temperatures measured near the flanges on the exposed and unexposed sides of CFS walls are different during standard fire tests. In these tests, wall specimens are mounted on a rigid frame and then directly exposed to fire on one side. The temperature difference between the expose and unexposed flanges changes over time (Fig. 1), thus the center of resistance is continuously changing as well as the strength and stiffness of CFS members.

Fig. 1 shows the temperature difference between the flanges of a 25-gauge Marino-Ware ViperStud during a standard fire test. This 0.015 in. thick stud is used in drywall systems, and the size of its web is 3.625 in. During the test, thermal gradients were relatively low until water started evaporating from the gypsum and the paper covering the exposed boards deteriorated, at about 15 minutes under fire exposure. At this stage, the average temperature on the flanges was 120 °C, and the temperature difference was 20 °C, approximately. Then, the thermal gradient rapidly increased, reaching a peak at 31 minutes, when the temperature difference between flanges was 244 °C. Later, the gradient decreased as the temperatures on the flanges approached 671 °C. At the end of the test, the exposed gypsum boards were deteriorated, and the joints between boards were slightly open.



Figure 1: Exposed and unexposed flanges temperature difference during a standard fire test

2.2 Member behavior under thermal gradients

In summary, non-uniform elevated temperatures produce degradation of mechanical properties, a shift of the center of resistance, and thermal deformations of the studs, including deflections out of the plane of the wall. Under these effects, the thin-walled studs develop a significant interaction of modes; therefore, it has been challenging for experimentalists to identify limit states.

Feng et al. (2003) tested 52 lipped channel columns (approximately 15.75 in. long) and visually determined failure modes. They reported 7 out of 16 specimens tested at ambient temperature exhibited interaction of modes at failure; while 33 out of the 36 specimens tested at uniform elevated temperatures clearly showed mode interaction. This implies the interaction of modes is more pronounced at high temperatures. Also, through computational analysis, Batista & Schafer (2013) showed the interaction of modes evolves as the temperatures increase and thermal gradients develop. Furthermore, Li et al. (2014) demonstrated that modal identification through the constrained Finite Strip Method could be used to characterize the coupling and switching of modes triggered by thermal gradients.

3. Modal Identification for Shell Finite Element

As has been typical of the *c*FSM literature (Ádány & Schafer 2006; Ádány & Schafer 2006; Li et al. 2014; Li & Schafer 2010; Li & Schafer 2009; Li & Schafer 2013), application of the mechanical assumptions for each mode class: G, D, L, or ST, defines a subspace of the original full deformation space that is limited to each mode class. Namely, *c*FSM defines constraint matrices, R_M , where M is G, D, L, or ST. The columns of R_M equate to a set of base vectors for

subspace *M*. Utilizing the combination of different shape functions related to boundary conditions, a generalized base functions are proposed in Li et al. (2011; 2013) to potentially handle any arbitrary boundary conditions. The formulated base vectors from cFSM needs an appropriate transformation for a shell FE model by interpolating the *c*FSM basis (R_M)_{FS} to the FE DOFs (R_M)_{FE}. Because general shell finite elements typically have 6 DOF, i.e., 3 translational and 3 rotational displacements, at each node while finite strips typically have 4 DOF, i.e., 3 translational displacements and 1 rotational displacement about the longitudinal axis. The details of the interpolation process can be found in Li et al. (2011; 2013). Finally, with (R_M)_{FE} known, any shell FE displacement vector, D_{FE} , can be approximated by a linear combination of the *c*FSM basis vectors interpolated to FE DOF through a linear least squares problem for the best approximation. Then, the contribution coefficients (for *G*, *D*, *L*, and *ST*) can thus be evaluated and participation of each mode class can be obtained.

4. FEM Modeling

The computational modeling using shell finite element of thin-walled structures for ultimate strength prediction and investigation of collapse behavior are highly sensitive a variety of model inputs, such as geometric imperfections, residual stresses, plastic strain, yield criteria, material model, boundary conditions, and also the fundamental mechanics, particularly with regard to element selection and solution schemes (Schafer et al. 2010). In addition, the simulation of collapse behaviors necessitates the inclusion of both geometric and material nonlinearity. For members subjected to elevated temperatures, the degradation of mechanical properties and thermal expansions need to be accurately considered. The temperature changes also need to be reflected in the model. All the analyses performed herein utilize the commercial finite element package ABAQUS (Simulia 2012). In this study, residual stresses and plastic strains are not included in this model. Details about the modeling parameters are following the studies in Schafer et al. (2010) and a brief summary is provided herein with expanded details on the material properties and thermal gradient. Additional model assumptions are discussed in details as follows.

- Mesh and elements: the shell element used here is the S4 shell element from the ABAQUS library of elements and the mesh is a medium fine model as can be seen in Figure 3 at the cross-sectional level.
- Boundary conditions: the end boundary conditions simulate local-plate simply supported conditions, which imply warping fixity at the member ends.
- Imperfections: for modeling convenience, the distribution of the imperfection are seeded from the local and/or distortional buckling mode shapes generated from a CUFSM analysis [7], and the magnitude is a function of the plate thickness.
- Pre-loading: a 10% of the nominal design axial strength of the member is on before the temperature loading

4.1 Temperature and thermal gradient

As shown in Fig. 1, the exposed and unexposed sides have a temperature difference during the fire. The thermal gradient between the two sides depends not only on the insulation and configuration of the wall but also the geometry of the cold-formed steel members itself related to the conductivity. However the measured temperature data is very limited. Thus, in this study, the tested data from US Gypsum (USG) on a SFIA 362S125-18 member (Batista 2015) was

employed as plotted in Fig. 2. The temperatures were measured on the exposed and unexposed flanges. The thermal gradient in this study was assumed to be linear as shown in Fig. 2a.



(a) Assumed thermal gradient in the memberb) temperature vs. time curve (T-t)Figure 2: Thermal gradient and Exposed and unexposed flanges temperature during a standard fire test

4.2 Material model

The material is assumed to be homogeneous and isotropic with Young's modulus E = 210,000 MPa, Poisson's ratio v = 0.3, and a yield stress of 345 MPa at the ambient temperature. The material properties at elevated temperature are summarized in Table 1.

E (MPa)	Poisson's ratio	T (°C)	α (1/°C)	T (°C)	σ_{true} (MPa)	$\epsilon_{\text{plastic}} (\text{mm/mm})$	T (°C)	σ _{true} (MPa)	$\epsilon_{\text{plastic}} (\text{mm/mm})$	T (°C)
203400	0.2879	20	1.22E-05	20	345.59	0		166.06	0	
189810	0.2901	100	1.28E-05	100	357.64	0.034227		180.22	0.001465	
172830	0.2929	200	1.36E-05	200	360.56	0.037342		194.62	0.003908	
145450	0.2957	300	1.44E-05	300	373.55	0.04207		209.39	0.007765	
117990	0.2986	400	1.52E-05	400	385.71	0.04678		224.74	0.013612	
90531	0.3021	500	1.60E-05	500	403.42	0.056166	20	240.92	0.022186	400
63073	0.3069	600	1.68E-05	600	418.9	0.065475		258.33	0.034397	
35615	0.3142	700	1.76E-05	700	432.21	0.074708		277.45	0.051338	
8156	0.3252	800	1.80E-05	750	443.78	0.083865		298.97	0.074271	
					453.95	0.092944		323.75	0.104589	
					463.25	0.101946		352.93	0.143745	
					471.47	0.110871		387.95	0.193147	
					479.01	0.119721		117.61	0	500
					486.15	0.128495		124.76	0.001695	
					492.69	0.137194		132.12	0.00473	
					498.8	0.14582		139.83	0.009917	
					504.54	0.154374		148.12	0.018465	
					509.96	0.162857		157.35	0.032119	
					515.14	0.171268		168.07	0.053286	
					519.99	0.179611		181.12	0.085107	
					524.85	0.187885		197.71	0.13147	
					529.18	0.196093		219.63	0.196765	
					193.76	0		76.05	0	600
					207.86	0.000995		79.66	0.001749	
					222.1	0.002464		83.44	0.005408	
					236.54	0.00455		87.56	0.012734	
					251.24	0.007428		92.38	0.026862	
					266.3	0.011305		98.51	0.053109	
					281.84	0.016423		107.08	0.099904	
					298	0.023063		120.13	0.179317	
					314.94	0.031541	350	141.17	0.306067	
					332.9	0.042209	550	43.23	0	
					352.13	0.055451		45.14	0.00382	
					372.95	0.071677		47.38	0.014237	700
					395.73	0.091315		50.48	0.041074	700
					420.91	0.114797		55.81	0.105881	
					449.02	0.142546		66.64	0.248618	
					480.66	0.174954		13.88	0	
					516.54	0.21237		18.14	0.17054	800
					604.4	0.303272		20.47	0.276918	

Table 1: Material properties of cold-formed steel

5. Numerical Studies

To illustrate the feasibility of the modal identification capability for members under the fire condition and demonstrate the potential complex coupled instabilities due to the thermal loading, three cold-formed steel members are selected as shown in Fig. 3. Note that these three members were used to highlight the capability of the modal identification for nonlinear collapse analysis under static loading in Li et al. (2013). These members were modeled as columns under the boundary conditions aforementioned. Based on the eigenvalue analyses, member (a) with a length of 600 mm is a local dominant model (first buckling mode is local buckling mode in the eigenbuckling analysis), member (b) with a length of 1200 mm is a distortional dominant model, and member (c) with a length 700 mm is a local-distortional interacted model (the first FEM eigenbuckling mode is a mixed local and distortional buckling mode). The nonlinear collapse analysis in Li et al. (2013) also highlighted the potential mode interaction and the interaction level depended on the imperfections. However, all the interactions were all local/distortional interaction sthrough slight global buckling interaction might happen for certain cases.



5.1 Member 1: local-dominated member

Modal identification of the nonlinear collapse behavior of this lipped channel under temperature loading is explored in this section. For member 1, three cases of imperfections were considered in the model: no imperfection (perfect geometry), a small local imperfection (0.1t), and a 50% percentage of imperfection based on Schafer & Peköz (1998) (i.e., a combination of 0.34t local and 0.94t distortional imperfection). Figure 4a provides the temperature-lateral deflection response at the middle of the column length, and Figure 4b provides the modal participations during the collapse of the column under the thermal gradient in Section 4.1. In addition, the nonlinear collapse behavior of the member with a 0.1t local imperfection under static axial length is also shown in Fig. 4c along with its modal identification in Figure 4d. Note, the shown load-displacement response for this axial loading case is the axial load vs. the end shorting and the results are re-plotted from Li et al. (2013). Moreover, the collapse modes of the member under thermal and static loading are also shown in Fig. 4e and f, respectively, for comparison.

Careful study of the overall response and evolution of the modal participations as a function of displacement, as provided in Fig. 4, reveals significant insights about the behavior of this member. First, significant lateral deflection at the monitored node in the middle of the column was developed under the thermal loading as shown in Fig. 4a. The thermal loading actually triggered a deflection in the opposite direction of the global direction in ABAQUS and then further developed to the positive direction with the increasing temperature. The large positive deflection can be seen from the collapse mode in Fig. 4e. However, the beginning negative deflection was more due to the local buckling because the monitored node is actually in the middle of web. Second, the mode participations in Fig. 4b of the member under the thermal gradient not only demonstrate the evolution of the buckling modes during the fire but also further corroborate the argument on the negative lateral deflection at the monitored node. Note that the participation at the beginning of the temperature loading reflects only the imperfection status as discussed in Li et al. (2013). There is a rapid growth of local buckling participation starting the temperature loading. Later, with the trigger of the weak-axis bending mode, which is part of the G modes, the participation of the G mode dominates while the local buckling participation decreases. However, there are two things worth noting: 1) the deformation goes through an interactive process between local and global modes; 2) even though the participation of local buckling, which is only the relative magnitude to other deformations, decreases, the actual local deformation indeed is increasing. Third, imperfection magnitude and distribution can alter the modal identification when the thermal load is small. When the thermal load gets large, their differences are small. Finally, compared to the static loading results, where the deformation and failure modes go through a local/distortional evolution and have a local buckling failure at peak and distortional buckling mechanism in the post-peak, the global buckling mode is more dominated with apparent contribution from local and distortional modes.



a) Temperature vs. deflection response at the midsection



b) Mode participation vs. the deflection



c) Load vs. end-shortening response (0.1t local imperfection)





d) Mode participation vs. the end-shortening



e) Collapse mode under thermal loading Figure 4: Nonlinear analysis and modal identification of Local dominated member

5.2 Member 2: distortional-dominated member

Modal identification of the nonlinear collapse behavior of this lipped channel under temperature loading is explored in this section. For member 2, similarly, three cases of imperfections were considered in the model: no imperfection (perfect geometry), a small distortional imperfection (0.1t), and a 50% percentage of imperfection. Fig. 5a provides the temperature-lateral deflection response at the middle of the column length, and Fig. 5b provides the modal participations during the collapse of the column under the thermal gradient. In addition, the nonlinear collapse behavior of the member with a 0.1t distortional imperfection under static axial length is also shown in Fig. 5c along with its modal identification in Fig. 5d. Moreover, the collapse modes of the member under thermal and static loading are also shown in Fig. 5e and f, respectively, for comparison.

The dominance of global deformations (i.e., weak-axis bending) in the collapse regimes is generally the same as in the local dominated model. In the beginning of the thermal loading, the member triggers a distortional buckling mode as evidence by the growth of the distortional participation in Fig. 5b though the growth is not visible for the 50% imperfection model because of the large presence of distortional mode already from the imperfection. The deformation then evolves into a global dominated mode though significant distortional mode is developed as can be seen in Fig. 5e. On the other hand, the member under the static loading demonstrates a consistent dominated distortional participation throughout the analysis as shown in Fig. 5d and visible in Fig. 5f.



a) Temperature vs. deflection response at the midsection



c) Load vs. end-shortening response (0.1t local imperfection)



b) Mode participation vs. the deflection



d) Mode participation vs. the end-shortening





5.3 Member 3: local-distortional interacted member

Modal identification of the nonlinear collapse behavior of this lipped channel under temperature loading is explored in this section. For member 3, similarly, three cases of imperfections were considered in the model: no imperfection (perfect geometry), a local distortional imperfection (0.1t), and a 50% percentage of imperfection. Fig. 6a provides the temperature-lateral deflection response at the middle of the column length, and Fig. 6b provides the modal participations

during the collapse of the column under the thermal gradient. In addition, the nonlinear collapse behavior and results of the member with a 0.1t local imperfection under static axial length are also shown in Fig. 6c, d, and e.

Similar to the previous two members, global deformations (i.e., weak-axis bending) in the collapse regimes is still the dominant mode. Similar to the member 2, in the beginning of the thermal loading, a distortional buckling mode is triggered as evidence by the growth of the distortional participation in Fig. 6b though the growth is not visible for the 50% imperfection model because of the large presence of distortional mode already from the imperfection. The deformation then evolves into a global dominated mode though significant distortional mode is developed as can be seen in Fig. 6e. On the other hand, the member under the static loading demonstrates a local and distortional interacted mode participation along with a more significant global mode participation, which is mainly weak-axis bending as well, as shown in Fig. 6d and visible in Fig. 6f.



a) Temperature vs. deflection response at the midsection



c) Load vs. end-shortening response (0.1t local imperfection)



b) Mode participation vs. the deflection



d) Mode participation vs. the end-shortening



e) Collapse mode under thermal loading f) Collapse mode under static loading Figure 6: Nonlinear analysis and modal identification of local-distortional interactive member

6. Discussions

The examples provided herein provide the feasible application of the modal identification to investigate the cold-formed steel member under thermal loading. While the numerical studies are limited, the demonstrated capability of the modal identification can potentially be widely applicable to track the more complex instabilities of cold-formed steel members under fire and identify the potential failure mechanism. Further studies towards the actual stud members with wall sheathing under fire are underway.

The computational models do not include the residual stresses that may be important to the behaviors of the cold-formed steel members under fire. Modal identification is still expected to be feasible even with the inclusion of residual stresses because the key of modal identification lies on the approximation of the general deformation field.

The thermal gradient studied herein is from the measured data on a specific member. As mentioned before, the cross-sectional geometry of the member has a large influence on the thermal gradient. An accurate thermal analysis may be needed to calculate the real distribution. Even by adopting simplified patterns, more analyses with an integration of the modal identification are necessary to investigate the impact of the thermal gradients. An ultimate goal is to determine how this quantitative modal identification approach can advance the analysis and design of the member under fire.

7. Conclusions

Modal identification of thin-walled members under thermal gradient modeled by shell finite elements and analyzed to collapse is successfully demonstrated. The modal identification method based on the constrained finite strip method can aid to track and identify the deformation modes of the member subject to elevated temperature loading during the fire. The application is demonstrated through a series of numerical examples of geometrically perfect and/or imperfect cold-formed steel lipped channel members under thermal gradient. Under thermal gradient, the strength and stiffness of the members are altered due to the degradation of mechanical properties and distributed unevenly in the member. The results indicate how the thermal gradient triggers a different deformation compared to the static loading and highlight the evolution of the deformation modes during the analysis by providing the participation of a given buckling-

associated deformation space (global, distortional, local, and/or shear and transverse extension) using the modal identification approach. Given the quantitative insights available with the modal identification approach explored here, significant future work remains to further utilize this capability to the advance of analysis and design of member under fire.

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