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Predicting the Buckling Strength of Spirally Welded Tapered Tubes Under Flexural Bending Using Reference Resistance Design

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Abstract
This paper covers the process of building capacity strength curves for medium length spirally welded tapered tubes (SWTs) under bending. The method used in this paper for building the capacity strength curves, is the Reference Resistance Design (RRD) as described in Eurocode 3 Part 1-6. The method allows an engineer with only a calculator to benefit from the complicated numerical analyses without going into the hassle of building finite element models. In this paper the flexural strength of 99 finite element models with variable slenderness ratio and imperfections amplitude are used to build the reference resistance design curves for SWT sections and validated with available SWT test results.

1. Introduction

For decades, the design of shells was based on knockdown factors obtained from test data for specific types of shells. The Eurocode 1993 Part 1-6-2007 for Strength and Stability of Metal Shells (EC3-1-6) was the first to adopt detailed guidelines for design using numerical modeling (finite element modeling). The design using numerical models is an exhaustive process that requires advanced knowledge of finite element modeling and the validation requires conducting a parametric study on geometric features and boundary conditions that could affect the final results and comparing the results to available test rest results to get what is referred to as a reliable model. European Code and National annexes adopted a more simplified method, developed by Rotter et al. (2011), the so called “Reference Resistance Design” (RRD) which allows a designer with a calculator to benefit from the whole computation power of finite element modeling without getting into the hassle of building the complicated models. The method involves building standardized capacity curves by running hundreds of validated finite element models of a specific shelled structures. Although the RRD method simplifies the design process, building the curves requires advanced knowledge of finite element modeling, studying the effect of imperfections pattern and amplitude, and validation with experimental results of a similar type of shells. A very similar

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approach in design is known in the United States and adopted by design guides for design of cold-formed steel members is known as the "Direct Strength Method".

2. Reference Resistance Design

EC3-1-6 described two methods in detail for the buckling limit state strength prediction of shells using numerical models, which is the base for building RRD curves. The first method is the LBA-MNA; where LBA is the Linear Bifurcation (or Buckling) Analysis, and MNA is the Materially Nonlinear Analysis. In this method, only the perfect geometry of the shell is required for creating the models (i.e. neither the imperfections nor the nonlinear geometry are considered). The LBA models are used to get the critical loads and stresses, and MNA models are used to get the plastic loads. The second method is a more complicated one, the Geometrical and Material Nonlinear Analysis with Imperfections implemented (GMNIA), which requires a set of linear and nonlinear analyses on both the perfect geometry and the geometry with initial imperfections, see Fig. 1. Design using global numerical GMNIA analyses is a sophisticated process that intends to use the full power of numerical modeling tools in design. The procedure requires running a set of analyses on several steps increasing sophistication of your model in every step and then testing and calibrating your final model by another set of analyses. RRD method performs this sophisticated GMNIA modeling in advance for a particular class of shells and then provides the output of those models as simplified capacity strength curves that the designer can utilize without performing their own analysis. RRD allows a designer to benefit from verified numerical analyses of a similar structure without going into the sophistication of building and verifying their own numerical models. To develop RRD, the procedure consists of running numerical models with varied slenderness ratio and imperfection amplitudes, verify the results with available test results or verified hand calculations, then building standardized strength curves using normalized results from numerical models. The argument of what type of imperfections to be implemented in the FE models to get reliable and distinct results and the verification of GMNIA models with the experimental results are discussed in Mahmoud et al. (2018). The choice of imperfection patterns is critical for creating RRD curves which is another discussion. The next section describes the process of building RRD curves for medium length SWT where the ovalization buckling mode is restricted and the local buckling is the critical limit state.

The Design using GMNIA as EC3-1-6 specified, requires several analyses with less details, than GMNIA, to act as upper bound to GMNIA models results. For each geometry, several models were created that varies in complexity and features, they are listed as follows:

1. LBA models: Linear Buckling Analyses are performed using the perfect geometry with nominal dimensions to get reference critical buckling moment M_{cr} and 1st eigenmode-affine patterns to use as an imperfection pattern (if there is no other reasonable imperfection pattern implemented).
2. GMNA models: Geometric and Material Nonlinear Analyses using the perfect geometry with nominal dimensions for computing the upper bound of the nonlinear analyses using the perfect nominal geometry of specimens.
3. GMNIA models: Geometric and Material Nonlinear Analyses with Imperfections included, to estimate the strength of SWT specimens.

For the GMNIA models the effect of the imperfection pattern and amplitude should be tested to ensure these imperfections in these models severe the strength.

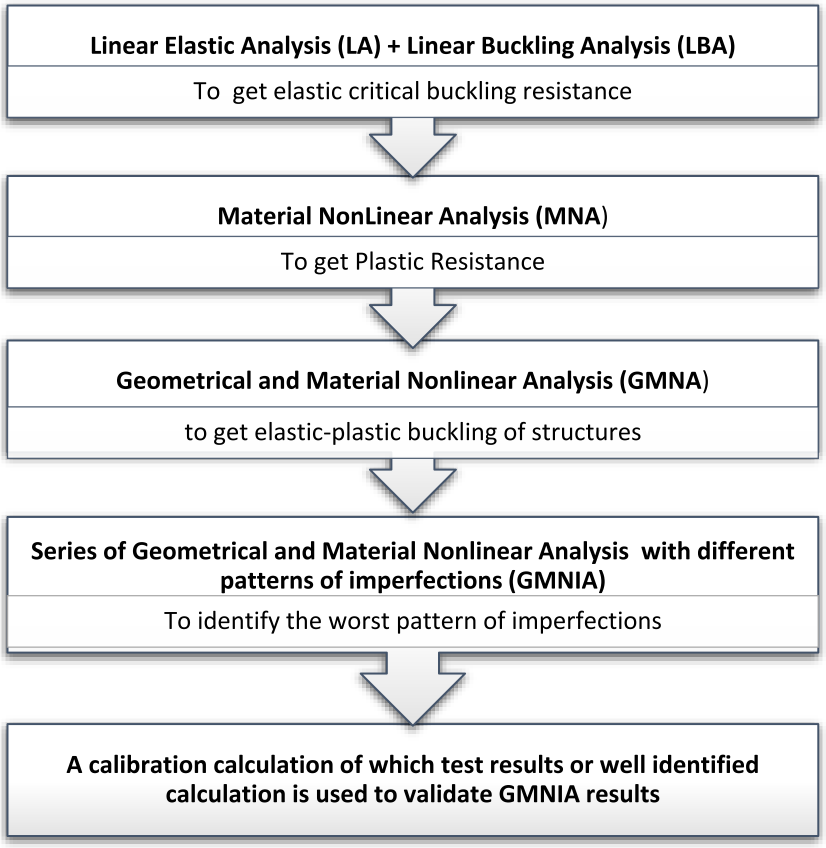


Figure 1: Design by GMNIA analyses according to EC3 (ECCS 2013)

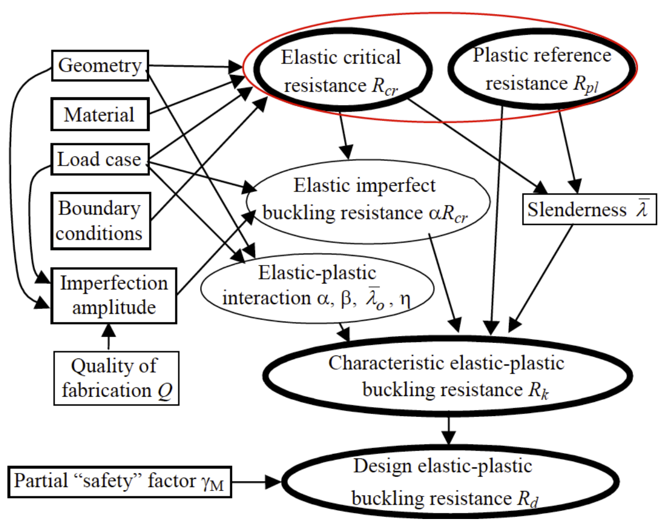


Figure 2: The processes of RRD method (Rotter 2016).

RRD curves are built to such that normalized GMNIA results, of a specific geometry of a shell under specific loading, can be used to predict the strength of a similar shell geometry. Due to the great variety of shapes and forms of shells structures, unique curves are built for each shell structure. Such curves can potentially save lots of time consumed by designer in building finite element models. EC3-1-6 section 8.5 provides guidelines for the buckling limit state in the design of shells using standardized capacity curves. Eq. 1-3 provides the capacity curves equations by calculating, the buckling reduction factor χ as the ratio of the GMNIA ultimate moment M_k to the plastic moment M_p, $\left(\chi = \frac{M_k}{M_p}\right)$, which is typically plotted against relative slenderness $\left(\lambda = \sqrt{\frac{M_p}{M_{cr}}}\right)$ or relative strength $\left(\frac{M_k}{M_{cr}}\right)$ as shown in Fig. 3. The elastic imperfection reduction factor (α), the interaction component (η), the plastic range factor (β), and the squash limit relative slenderness (λ_o), these parameters (RRD parameters) are left to designers to be computed for a specific structures RRD curves.

$$\chi = 1 \qquad \text{when } \lambda \leq \lambda_o \qquad (1)$$

$$\chi = 1 - \beta\left(\frac{\lambda - \lambda_o}{\lambda_p - \lambda_o}\right)^{\eta} \qquad \text{when } \lambda_o < \lambda < \lambda_p \qquad (2)$$

$$\chi = \frac{\alpha}{\lambda^2} \qquad \text{when } \lambda_p \leq \lambda \qquad (3)$$

where the plastic limit slenderness $\left(\bar{\lambda}_p = \sqrt{\frac{\alpha}{1-\beta}}\right)$.

The RRD parameters ($\alpha, 1-\beta,$ and η) are represented in the form $\left(\frac{a}{1+b(\delta_o/t)^c}\right)$ where $a, b,$ and c are the constants to be computed for every parameter from GMNIA models.

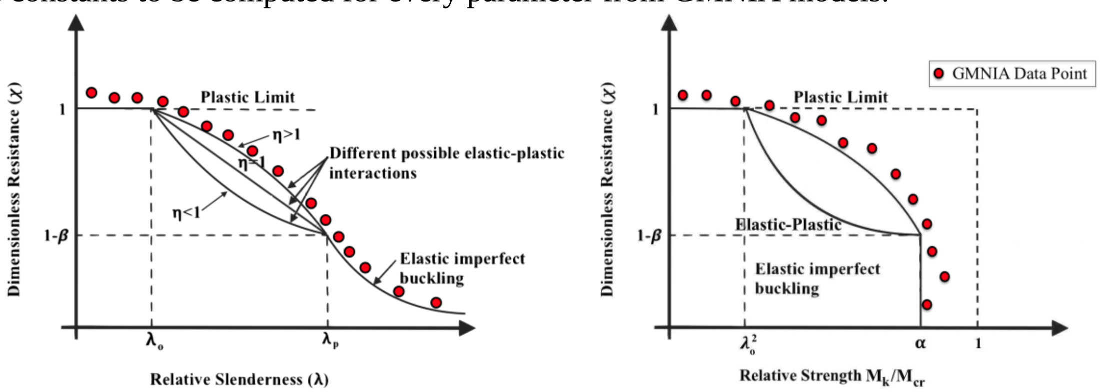


Figure 3: Typical shell strength curves (Mahmoud 2018).

3. RRD for Spirally Welded Tapered Tubes

Spirally Welded Tapered Tubes (SWTs) have special geometric features and imperfection patterns. The RRD capacity curves should be uniquely built for a specific geometry of a shell structures under specific load, the following sections describes building and calibrating RRD curves specifically for SWTs. The results from GMNIA models were validated with the available large-scale SWT experimental results provided by (Jay et al. 2016) in previous studies (Mahmoud et al. 2015, 2016, 2018).

3.1 Parametric model

For the scope of this study where the failure is controlled by local buckling and not the ovalization mode, a shell numerical parametric model was created for building the RRD curves. This parametric model was chosen to be close to the SWT specimens (reported by Jay et al. 2016) that will be used to verify the proposed RRD curves. A SWT model with maximum diameter (D_{max} =

1000 mm), the minimum diameter (D_{min} =860 mm), and the length of the model is ($L = 4D_{max} =$ 4000 mm), as sketched in Fig. 4. The width of the plates that form the tube is 300 mm, which will result in a range of helical angles from 5.5° at the largest diameter to 6.3° at the smallest diameter, as the helical angle changes with the variation in diameter along the length, see Fig. 4.

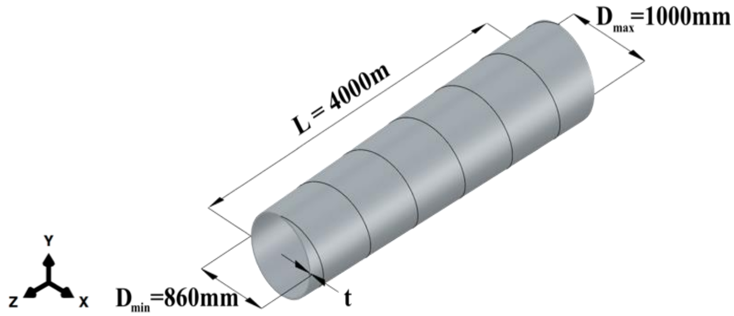
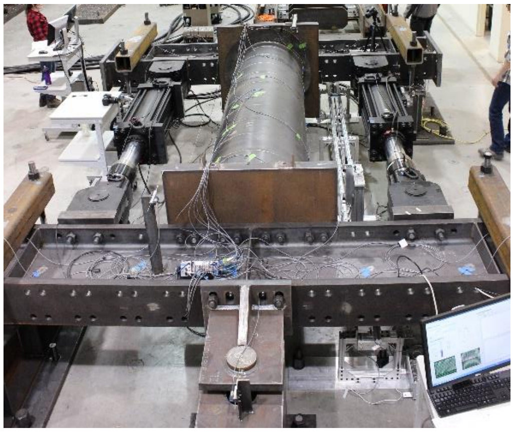


Figure 4: Spirally welded tube parametric model dimensions (Mahmoud 2018).

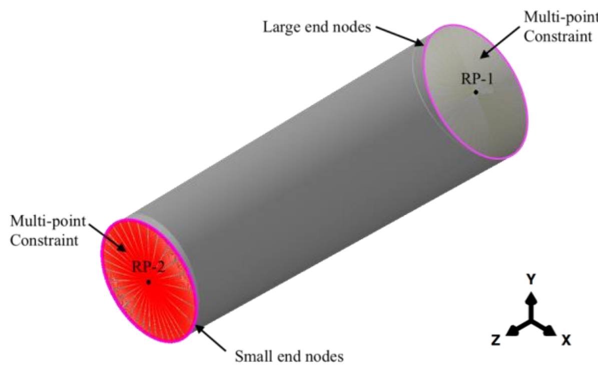
For the same geometry eleven models were created with thicknesses ranging from 40 mm to 2.86 mm, these thicknesses are selected to represent a range of maximum radius-to-thickness ratios from 25 to 350, to represent the relative slenderness (0.35 to 1.35), as the relative slenderness of SWT tests used for verification were between (0.54 and 0.93). The material model used is an elastic perfectly plastic model with elastic modulus of 200 GPa, yield stress of 460 MPa, and ultimate strain of 0.15.

3.2 Boundary Conditions

The boundary conditions of the numerical models are selected to be consistent with those for the SWT experiments, Fig. 5(a), which was used to validate the GMNIA models. Using two beam-type Multi-Point Constraints (MPC), one at each end of the tube, connecting all the nodes at each end to a reference point that coincide with the center point of the cross section at this end, see Fig. 5(b). This type of constraint prevents any relative deformation between the reference point and the end nodes of the model, thereby restricting ovalization of the ends, and permits all end nodes along with the reference point to collectively displace or rotate in any direction. The loads and boundary conditions are then applied to these reference points. The boundary conditions applied to these two reference points are defined in Table 1. The applied boundary conditions allow flexural end rotations at both end, restrict in-plane displacements across the member (i.e., in the X or Y directions) at both ends, restrict meridional (longitudinal) displacements (i.e., in the Z direction) at the small diameter end (RP-2), allow meridional displacement at the large diameter end (RP-1), and restrict torsional rotations at both ends. Flexural loading is considered with two equal end moments (M_x) applied to both ends.



(a)



(b)

Figure 5: (a) a picture of the test rig showing the boundary condition at the end of the specimen (Jay et al. 2016) , (b) Multi-point constraint (Mahmoud et al. 2016).

Table 1: Restraints of reference points.

Reference Point Name	Location	Displacement			Rotation		
		X	Y	Z	X	Y	Z
RP-1	Large diameter end	X	X				X
RP-2	Small diameter end	X	X	X			X

3.3 Elastic Critical Moments and Plastic Moments

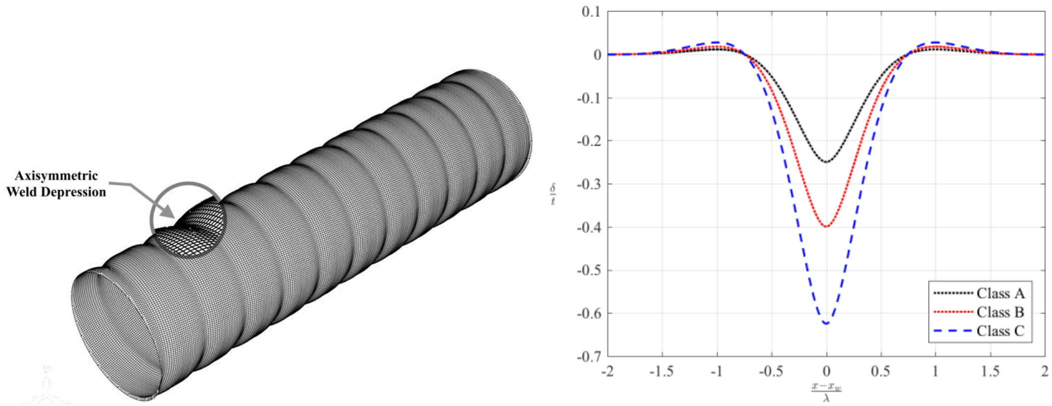
The analyses required to get the plastic resistance and elastic critical resistance could be substituted by theoretical solutions, to save computation time. In a previous study by the authors, modeling protocols were established to provide results from numerical models that converges to the theoretical solutions (Mahmoud et al. 2018). The critical moments from LBA runs, and plastic moments from MNA runs, could be substituted with theoretical critical moment (Eq. 5) and plastic moments (Eq. 4). The theoretical plastic moment (Eq. 4) are computed using equivalent section with radius $R_{eq} = \left(\frac{R_{max}+R_{min}}{2\cos\phi}\right)$, where ϕ is the angle of tapering.

Plastic moment for thin-walled shells $$M_p = 4R_{eq}^2 t\sigma_y \quad (4)$$

Critical moment $$M_{cr} = \pi\left[\frac{ER_{min}t^2}{\sqrt{3(1-\upsilon^2)}}\right]\cos^2\phi \quad (5)$$

3.4 Imperfections Pattern

For building the GMNIA models necessary for RRD, two approaches for generated imperfections patterns were tested. The first approach is to use the lowest eigenmode-affine pattern, obtained from LBA of the perfect geometry of the model, as your imperfection pattern, although this approach is easier, the results of GMNIA models with different imperfection amplitudes were not as distinct as it was expected to be (Mahmoud et al. 2018). The second approach, is to use a weld depression imperfections pattern of Rotter and Teng (1989), "Type A", as shown in Fig. 6(a). This weld depression profile is applied along the spiral seam welds on the parametric model. This approach is proved to be more suitable for creating the curves and the results of the GMNIA models are more distinct when varying the imperfection pattern amplitude. In this case, the imperfection profile amplitude (δ) is scaled to the thickness of each model: (0.05t, 0.1t, 0.2t, 0.4t, 0.6t, 0.8t, 1t, 1.5t, and 2t) (note that the imperfection amplitudes according to EC3-1-6 quality classes (Class A=0.25t, Class B=0.4t, and Class C=0.625t), see Fig. 6(b).



(a) (b)

Figure 6: (a) Typical spirally welded tube with scaled weld depression imperfection along spiral seam weld, and (b) weld depression profiles as recommended by EC3 using Type A weld profile proposed by Rotter and Teng (1989) (Mahmoud 2018).

3.5 RRD Flexural Capacity Curves for SWTs

The models were built with constant geometry (maximum diameter, minimum diameter and length), but with varied thickness to match the maximum radius-to-thickness ratios and amplitude of the imperfection pattern. Fig. 7 shows an example of the moment rotation curves for one set of the GMNIA models with radius-to-thickness ratio ($\mathrm{R/t} = 200$) and amplitude of imperfection pattern ($\delta_o = 0.05t - 2t$). The results are shown in Fig. 9-10, where each curve connects the same imperfection magnitude for all the models with different slenderness. Fig. 8 shows the results in terms of normalized moment and relative slenderness. Notice the strength of the models with lowest slenderness ratio drops below the plastic moment. This drop gets more severe to models with higher slenderness and the same imperfection amplitude, as the imperfection amplitude for low slenderness tubes are large (in this case when $\mathrm{R/t} = 25$, the thickness is 20 mm and the amplitude for imperfection profile range from 1 mm to 40 mm). Such an imperfection provides numerical convenience but is not practically important. In the models with high slenderness, where elastic buckling is the controlling limit state the effect of the imperfections decreases and the strength curves get closer to each other. The slenderness of the SWT tests reported by Jay et al. (2016) lies in the elastic-plastic range, the effects of low and high slenderness ratios are not as important for the scope of this study, the imperfection amplitude reported in all specimens, except one outlier, were in the range ($0.4t < \delta < 0.625t$) which is the quality class C requirement in EC3-1-6. The outlier test was classified as worse than class C (WTC).

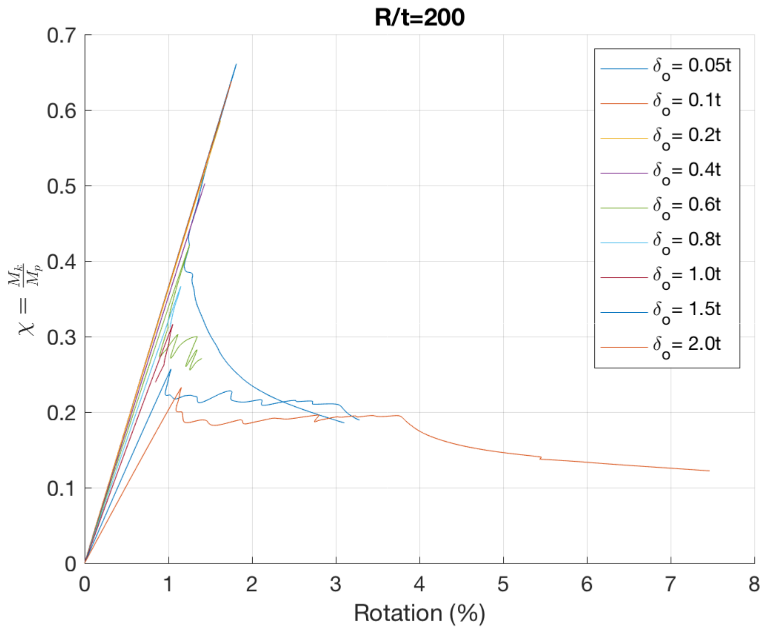


Figure 7: An example of the moment-rotation curves for GMNIA models with R/t=200.

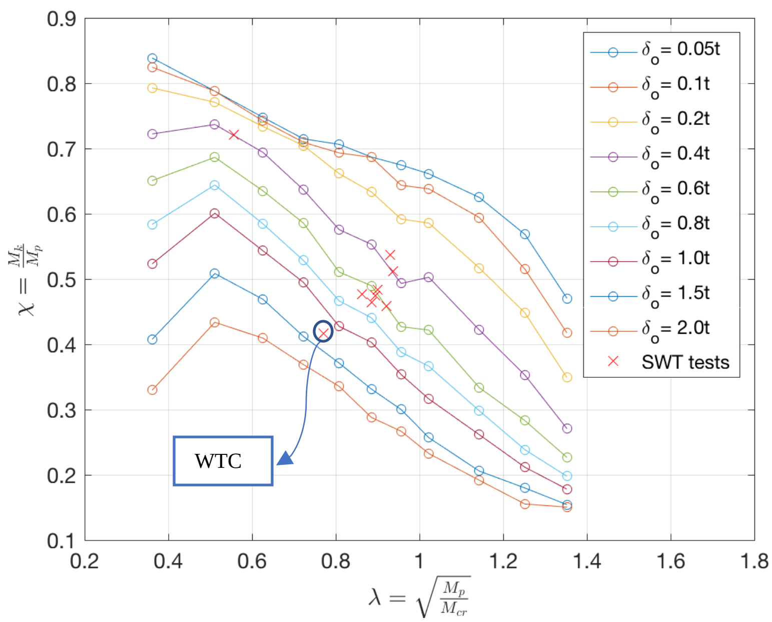


Figure 8: Capacity curves of normalized flexural strength of GMNIA models of spirally welded tapered tubes in terms of relative slenderness.

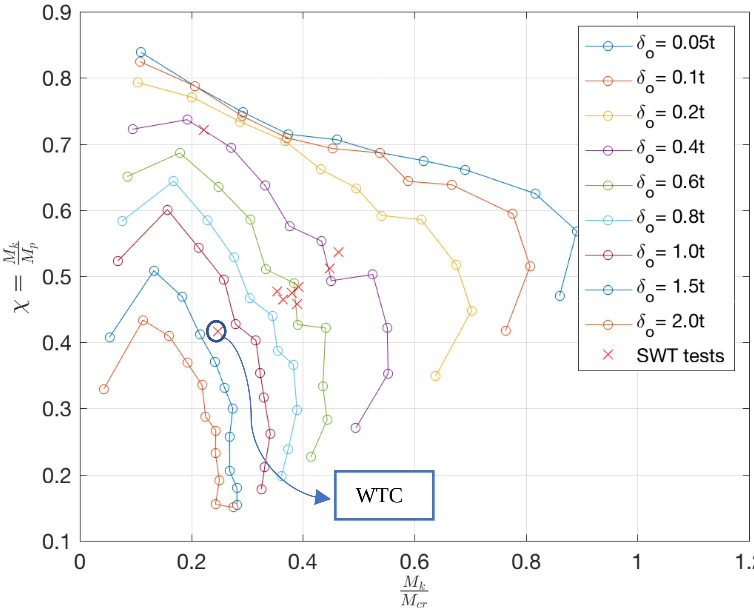
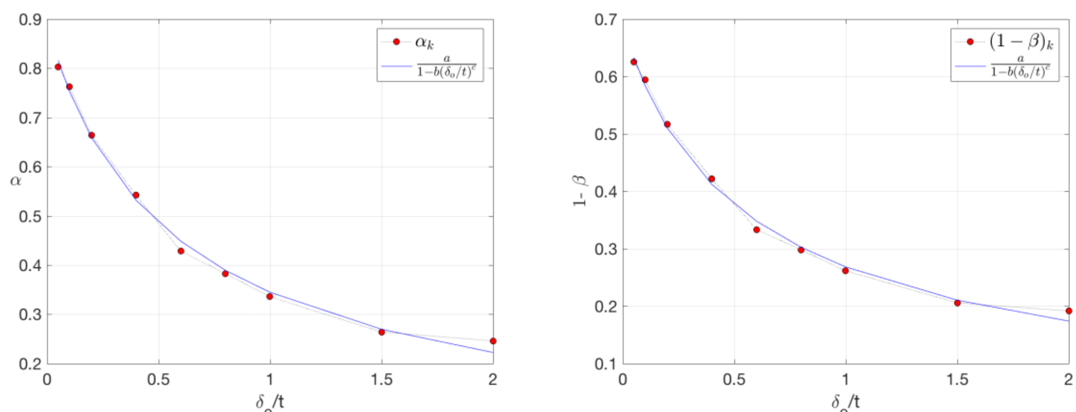


Figure 9: Capacity curves of normalized flexural strength of GMNIA models of spirally welded tapered tubes in terms of relative strength.

3.6 RRD parameters for SWTs curves

The RRD parameters (α, β, and η) are established to match the RRD curves built with GMNIA results for the SWT parametric models.

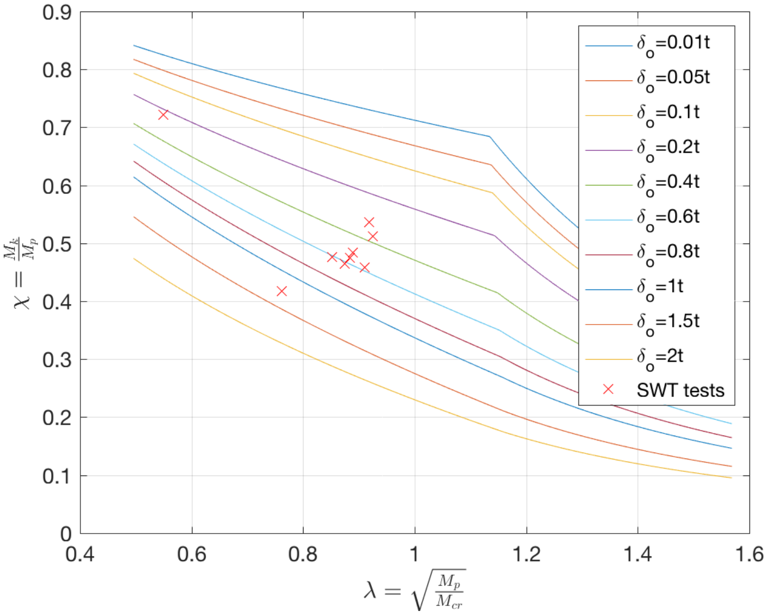


(a) (b)

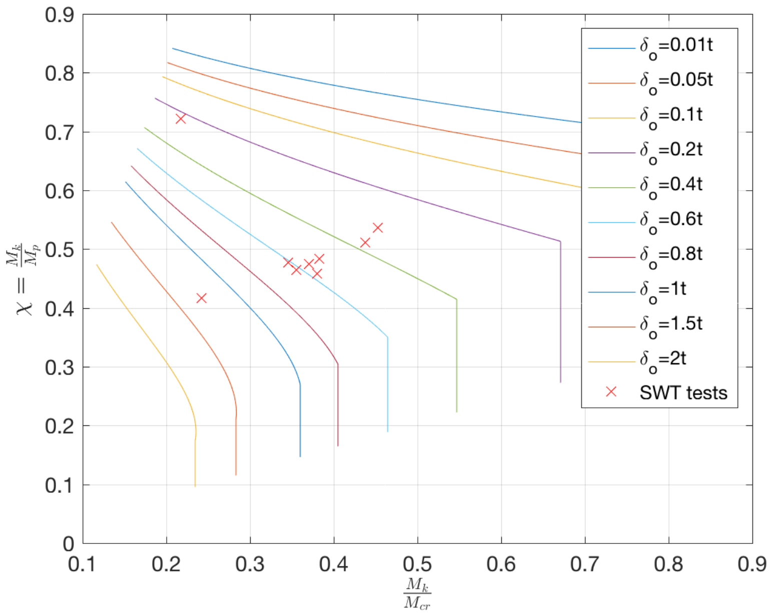
Figure 10: RRD curve parameters (a) elastic imperfection reduction factor, and (b) plastic range factor.

The fitted curves of the elastic imperfection reduction factor (α) and the plastic range factor (β), and the best fit to the interaction component (η) to minimize the differences between GMNIA models and the curves, are computed as:

$$1-\beta = \frac{0.70}{(1+1.6(\delta_o/t)^{0.92})}, \eta = \frac{0.60}{(1+0.1(\delta_o/t)^{2.5})} \text{ and } \alpha = \frac{0.90}{(1+1.5(\delta_o/t)^{0.92})}$$



(a)



(b)

Figure 11: Capacity curves of SWT built with updated RRD parameters with SWT tests results against (a) relative slenderness, and (b) relative strength.

4. Summary and Conclusions

In this paper, the Reference Resistance Design (RRD) method is described and used to create capacity strength curves for Spirally Welded Tapered Tubes (SWT) under flexural bending. The curves were built using geometrically and Materially Nonlinear Analysis with Imperfections (GMNIA). The GMNIA models used here was extensively studied by authors and validated with test results. The method is intended to simplify the process of predicting strength of a specific shell structure from the GMNIA models to the extent that an engineer with a calculator can predict strength of a shell structure. The parameters of RRD curves for SWT were established and the curves were built using these parameters. The RRD parameters established here can be used by designers to predict flexural strength of medium length spirally welded tapered tubes.

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