Proceedings of the
Annual Stability Conference
Structural Stability Research Council
St. Louis, Missouri, April 2-5, 2019

Lateral stability and design of Gerber systems

Amir Elmaraghy[1], Kévin Silva, Valentin Manaud[2], Nicolas Boissonnade[3]

Abstract

The present paper addresses the behaviour, resistance and design of steel beams with overhanging segments against Lateral Torsional Buckling – so-called Gerber systems. This system has been used extensively in North America for the roof girders of single story buildings – typically warehouses and commercial buildings. This system is quite rational and economic, and also requires simple fabrication and erection works. However, the design process deserves careful and specific attention with respect to (i) the L.T.B. check of the overhanging segment which cannot be assumed as a typical cantilever since the adjacent back-span cannot provide full fixity, (ii) the L.T.B. check of simply supported parts under realistic lateral restraints and (iii) the design of connections at vertical supports, namely with respect to bracing considerations.

Research investigations aimed at understanding deeper the buckling behaviour of such systems, characterizing the key design parameters and developing an adequate design method were undertaken and are reported in the paper. Comprehensive F.E. studies on various major influences such as span ratios, lateral support conditions or the need for vertical stiffeners are detailed and analysed. Eventually, a devoted set of design rules and recommendations is proposed.

1 Introduction – Context and objectives

The present paper addresses the behaviour, resistance and design of steel beams with overhanging segments against Lateral Torsional Buckling (L.T.B.). Such structural elements are quite popular in North America and Canada, and used widely in so-called "Gerber systems" for multi-bay arrangements, cf. Figure 1. Usually, they are used as framing systems for roofs and in particular for large floorplate single storey buildings.

This system is also widely used across Europe, with different materials (concrete, steel, timber) and for various types of girders – from purlins to bridges. It has the advantages of maintaining a seemingly indeterminate pattern of bending moment distributions – thus leading to effective and economic balance of hogging and sagging bending moments as well as to reduced deflections –, while avoiding complex and costly moment connections. It is indeed possible to select carefully the regions where simple, shear-only joints are placed, so that they lie close to

[1] Graduate Student, Laval University, <amir.elmaraghy.1@ulaval.ca>
[2] Graduate Student, Laval University, <valentin.manaud.1@ulaval.ca>
[3] Professor, Laval University, <nicolas.boissonnade@gci.ulaval.ca>

the natural zero-moment sections of continuous beams, nearly recreating the natural continuous beam bending moment distributions.

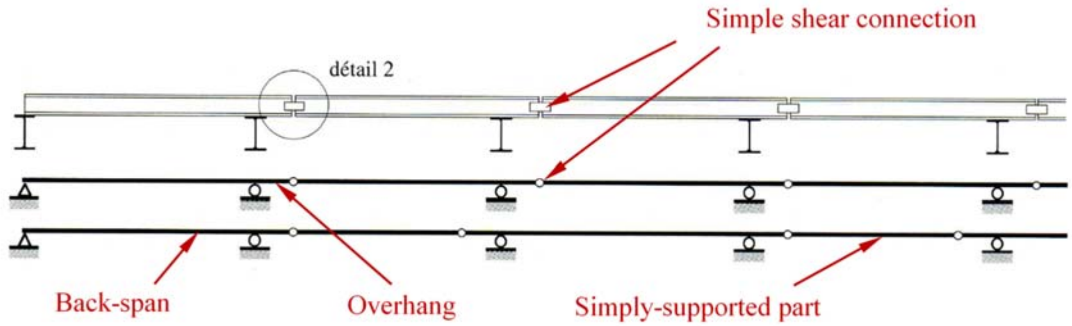


Figure 1: Typical Gerber systems (from Hirt et al., 2016)

In practice, either Figure 1 static system #1 (identical girders with overhang segment) or #2 (with shorter simply supported parts) are met. Both require a similar number of connections and lead to equivalent bending moment distributions.

The system is quite rational and economic, and also requires simple fabrication and erection works. However, the design process deserves careful and specific attention with respect to:

- The L.T.B. check of the overhanging segment, which cannot be assumed as a typical cantilever since the adjacent back-span cannot provide full fixity, in particular regarding weak axis and warping restraints. These are crucial issues regarding the L.T.B. resistance of the overhanging segment, and they are unfortunately poorly addressed by designers, potentially leading to unsafe resistance predictions. The latter may be even aggravated by the fact that the bottom flange under compression in the zone of hogging bending moments usually cannot expect any lateral support from the roof elements;
- The L.T.B. check of simply supported parts (cf. Figure 1): the segment between points of zero moment (simply supported determinate segment) is usually designed on the assumption that fork support conditions apply. This is incorrect and unsafe, as a clear lack of lateral horizontal support is brought by the overhanging segments. Dedicated guidance on the determination of the effective L.T.B. length is here clearly desirable;
- The design of connections at vertical supports: the horizontal lateral forces and torsional moments at points of peak hogging moments deserve adequate supports, and the current design provisions in Canada are insufficient.

Feedback from practitioners and designers show quite different practices and interpretations of the system, namely regarding joint detailing, the use of vertical stiffeners and details of lateral bracing:

- Vertical stiffeners may be omitted in girders' webs at columns;
- Possible bottom chord extension of secondary joist at column allowing lateral bracing of girder's bottom flange;
- Joist providing top flange lateral bracing at hinge location or not;
- Vertical stiffeners at hinge, either on one side or on both;
- Bottom flange lateral bracing at hinge;
- Lateral bracing at hinge top flanges in both girders…

These points cause practical issues on both design and construction stages, and can even lead to dramatic issues such as the accident in Barnaby, BC (2017). Accordingly, this paper intends at detailing research investigations directed towards a deeper understanding of the behaviour of such systems, at characterizing the key design parameters, and at developing an adequate design method. Following section 2 briefly sums up the current design guidance from the Canadian Standards; Section 3 then details shell non-linear F.E. models that have been used as references in extensive numerical parametric studies, whose results are presented and analysed along § 4. Eventually, Section 5 proposes new design recommendations, which are assessed against the numerical reference results and compared to existing design rules from major design codes.

2 Current design provisions in Canadian Standards

2.1 Design rules for Lateral Torsional Buckling

Currently, the Canadian Code for Steel Structures CSA S16 (CSA, 2014) mostly provides design guidelines for regular beams in its paragraphs 13.5 for laterally supported members and 13.6 for laterally unsupported members. In particular, § 13.6 defines "segments between effective brace points", and offers design rules for such segments, either for sections with doubly symmetric shapes or for mono-symmetric sections, as a function of the cross-section class (Class 1 and 2: plastic resistance; Class 3: elastic resistance; Class 4: effective resistance).

No clear definition of an "effective brace point" is given, and in particular whether this bracing is relative to both top and bottom flange lateral support or to only one of these. This question is particularly relevant in the Gerber system arrangements considered here, since both the length of the considered segment and its – lateral – support conditions are known to significantly affect the member's response and resistance to L.T.B.

S16 § 13.6 (CSA, 2014) relies on Timoshenko's basic formula for its design recommendations against L.T.B. and further introduces a factor ω_2 to account for the bending moment distribution along the girder (M_u designates the *unbraced* length critical moment and corresponds to the usual notation M_{cr}):

$$M_u = \frac{\omega_2 \pi}{L}\sqrt{GJ \cdot EI_y\left(1+\frac{\pi^2 EC_w}{L^2 \cdot GJ}\right)} \tag{1}$$

With:
$$\omega_2 = \frac{4M_{\max}}{\sqrt{M_{\max}^2 + 4M_a^2 + 7M_b^2 + 4M_c^2}} \tag{2}$$

Where M_a, M_b and M_c represent the values of the moments at the quarter, half and third quarter of the considered segment, respectively. As for the influence of a transverse loading being applied above the shear centre, only rough and over-safe recommendations are given: (i) use of $\omega_2 = 1.0$ (i.e. fictitiously consider the worst bending moment distribution) and (ii) exaggerate the segment's length by 20 to 40%, depending on the bracing conditions.

CSA S16 prescribes the ultimate resistance to L.T.B. M_r following 3 distinct parts: the first one is where instability does not affect the resistance of the beam (horizontal "plateau"), the second is a transition one (inelastic L.T.B.) and the third one corresponds to resistance being mostly governed by elastic L.T.B. These 3 different parts can be encapsulated as follows:

If $M_u > 0.67 M_p$
$$M_r = 1.15 \cdot \phi \cdot M_p\left[1 - \frac{0.28 M_p}{M_u}\right] \le \phi \cdot M_p \tag{3}$$

Else:
$$M_r = \phi \cdot M_u \tag{4}$$

Where M_p is the plastic bending resistance if the section is Class 1 or 2, $M_{elastic}$ for class 3 and $M_{effective}$ for class 4.

2.2 Particular case of Gerber systems

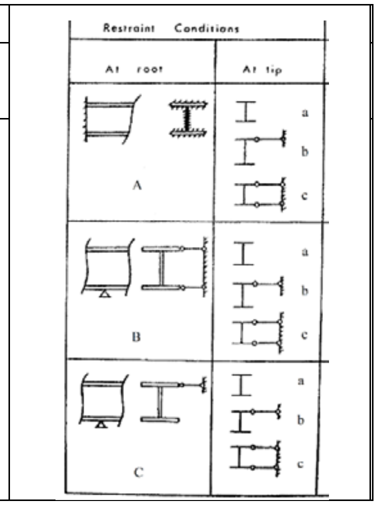
As previously stated, S16 lacks guidelines in the particular case of Gerber systems, especially regarding the cantilever part. Beaulieu et al. (2003) provide additional specific design recommendations to account for various support conditions on cantilever parts through Eq. (5):

$$M_{ue} = \frac{\pi}{KL}\sqrt{GJ \cdot EI_y\left(1+\frac{\pi^2 EC_w}{K^2L^2 \cdot GJ}\right)} \tag{5}$$

Where K depends on the support conditions, as given in Table 1.

Table 1: Coefficient K values as a function of cantilever support conditions

Lateral support conditions		Coefficient K	
Supported extremity	Unsupported extremity	Loads on the upper flange	Other cases
A	a	1.4	0.8
	b	1.4	0.7
	c	0.6	0.6
B	a	2.5	1.2
	b	2.5	1.5
	c	1.5	0.8
C	a	7.5	3.0
	b	7.5	2.7
	c	4.5	2.4



3 Research methodology – Development of suitable F.E. models

3.1 Research methodology

Owing to the many parameters susceptible to have an important influence on the systems' response, it appeared practically impossible to investigate the current questions experimentally. Therefore, decision has been made to rely on carefully-conducted F.E. numerical results, that shall serve as reference data to (i) understand the response of Gerber systems, (ii) identify and isolate the key factors influencing the resistance to bending, and (iii) assess a new, dedicated design method.

Accordingly, the research methodology described in the following basically consisted in:

- Developing and validating F.E. numerical models able to reach ultimate loads as accurately as experimental tests would (see next paragraphs);
- Collect numerical results investigating the influences of key parameters so as to understand the systems' behaviour (§ 4);
- Propose and assess an appropriate design methodology through systematic comparison with numerical reference results and suggest appropriate changes in CSA S16 dedicated clauses (section 5).

3.2 Shell F.E. models

All numerical simulations were performed by means of F.E. software FINELg (2012), continuously developed at the University of Liège and Greisch Engineering Office since 1970. For these studies, FINELg was used to perform Materially Non-linear Analyses (M.N.A), Linear Buckling Analyses (L.B.A) and Geometrically and Materially Non-linear with

Imperfections Analyses (G.M.N.I.A). Quadrangular 4-nodes plate-shell finite elements with Corotational total Lagrangian formulation according to Kirchhoff's theory for bending were chosen. L.B.A. calculations (i.e. critical load calculations) resorted to the so-called subspace iteration method. G.M.N.I.A. analyses were based on state-of-the-art numerical techniques and strategies: pure Newton-Raphson iterative scheme with out-of-balance residuals corrections, associated with the arc-length method and automatic loading strategies up to peak loads and beyond. Particular attention was paid guaranteeing that all peak loads kept as reference results were reached smoothly in the vicinity of the maximum load.

Material laws were carefully selected and implemented in the model. They follow the recent recommendations from Yun et al. (2017), and are based on the results of thousands of tensile tests. The yield plateau and strain hardening effects are included using a quadrilinear stress-strain relationship. In general, reference to 350W steel was made with characteristic values as follow: Young's modulus $E = 200$ GPa, yield stress $F_y = 350$ MPa and ultimate stress $F_u = 450$ MPa.

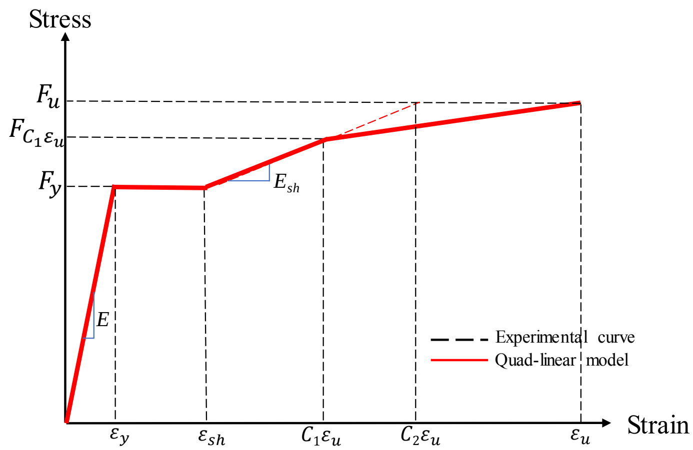


Figure 2: $\sigma - \varepsilon$ material law used in FE simulations (350W steel)

Section geometries and system arrangements primarily relied on the use of shell elements; possibilities to consider both hot-rolled and welded profiles were included, and several modelling specificities are further described next. First, the web-to-flange zone of hot-rolled sections received a specific treatment (see Fig. 3). Within shell modelling, this region suffers from (i) an overlap of material and from (ii) the absence of filets. This particularly influences the torsional response of the section and, by extension, resistance to L.T.B.

In order to get closer to the real characteristics of such steel sections, an additional node has been placed within the web height, at the exact vertical position of the centroid of the radius zone. In addition to being linked with the shell elements of the web, this node bears an additional beam element, oriented in the longitudinal direction, whose cross-section area is equal to that of the radius zones minus the overlapped area. Moreover, the section of this additional element is chosen to be a square hollow section, with height and thickness carefully adjusted to provide nearly-exact cross-sectional properties of the shell element in comparison to analytical ones, especially with respect to the torsional inertia. These beam elements naturally bare the same material constitutive law as the shell elements.

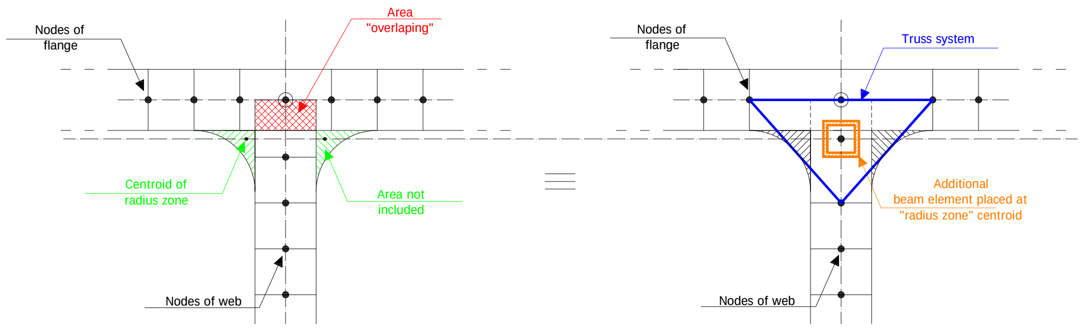


Figure 3: Modelling principles of web-to-flange area

Another improvement was brought by the introduction of additional truss elements to maintain the rigidity of the area influenced by the flange radius, i.e. to restrain local buckling at the web-to-flange junction. These truss systems were composed of 3 elastic elements with increased stiffness, creating rigid triangles.

As for welded sections, no such modelling refinements have been used, since the influence of the weld is much smaller and has therefore been deemed negligible.

3.3 Modelling of systems – Support conditions and loading

In order to separate the effects of in-span behavior from those of the cantilever, 3 different systems have been considered and modelled – see Fig. 4 to Fig. 6:

- System I represented the basic case of an isolated beam with fork end conditions and constant transverse loading applied at the top flange. It characterized the behavior and response of a typical drop-in segment under assumed under fork support conditions. System I has also been used to simulate the response of the edge spans, as acted by a transverse load combined with a negative bending moment arising from continuity with the cantilever parts;
- System II considered a cantilever segment of length L_B adjacent to the simply supported span L_A (Fig. 5). It aimed at accounting for the influences of (i) a non-fully rigid situation for the cantilever at support 2 and of (ii) a hogging bending moment on the resistance and stability of the back-span segment;
- System III simulated the full Gerber arrangement. Possibilities for various relative lengths L_A / L_B / L_C, 3 different cross-sections CS1 to CS3 and load levels q_A, q_{B+C+D} or q_E were used in further parametric studies.

All systems accounted for fork support conditions at their extremities, except system II at the free end of the cantilever. Intermediate supports may or may not have bottom and/or top flange lateral bracings.

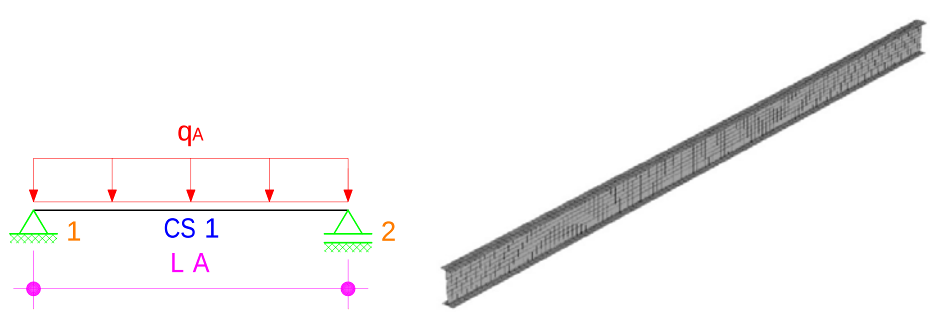


Figure 4: System I: simply-supported beam under fork conditions

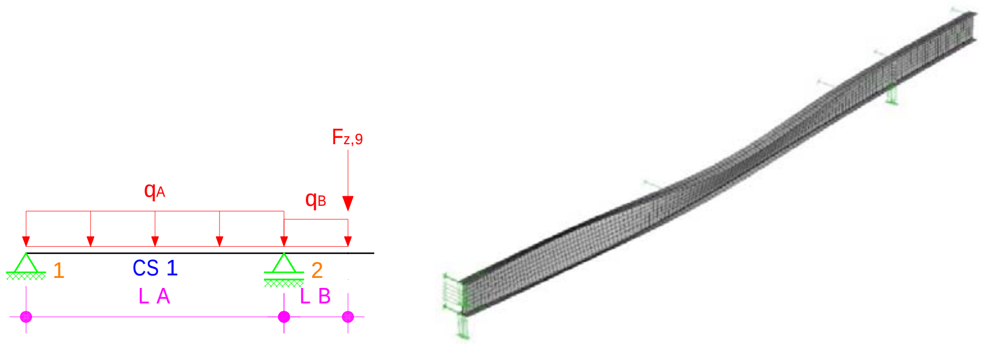


Figure 5: System II: simply-supported beam (fork conditions) with overhang segment

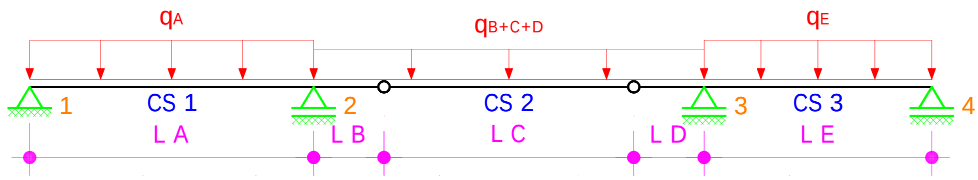


Figure 6: System III: full 3-span Gerber arrangement

Hinges have been modelled by joining a certain portion of the webs of the two beams' edges – see Fig. 7a. Coupled with various support conditions choices, it could be shown that they drastically influenced the L.T.B. response of girders – cf. § 4.7.

As for support conditions in system I, two main aspects have been distinguished for the definition of the reference "fork conditions" at the member's ends. The first one concerned the treatment of in-plane cross-sectional local supports. These have been defined as Fig. 7b shows, and consequently provided (i) local lateral support to possible local buckling owing to concentrated support reactions, as well as ii) global cross-section fork condition supports, namely lateral and vertical deflections, as well as torsional twist.

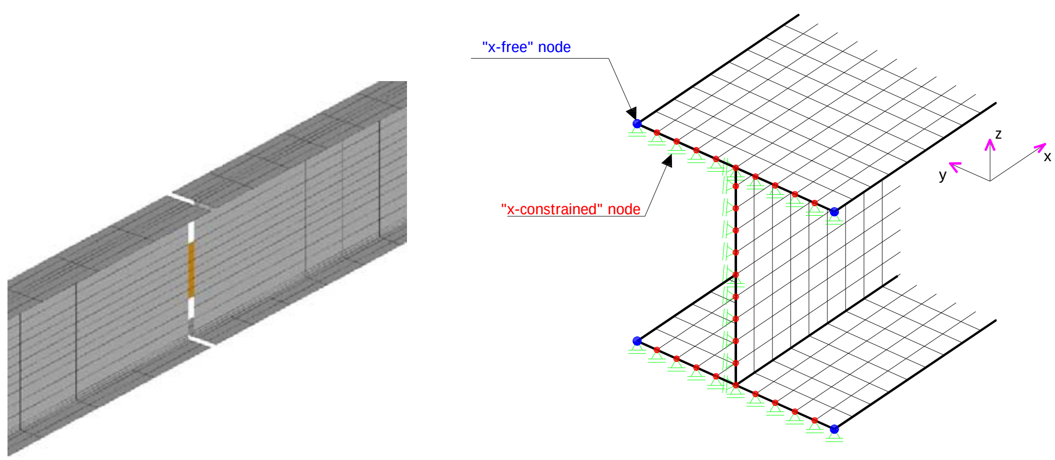


Figure 7: a) Numerical modelling of hinges – b) Modelling of end sections: transverse supports and linear constraints (longitudinal)

The second aspect dealt with possible axial displacements ("*x*-oriented") of the end cross-section nodes. In order to allow for a maximum number of four global degrees-of-freedom of the end cross-section (i.e. axial displacement, rotations θ_y, θ_z and warping), linear constraints have been used between the flange and web nodes. While a maximum of four nodes may experience a "free" longitudinal displacement, all other nodes' *x*-displacements linearly depend on the longitudinal displacements of the "*x*-free" nodes to respect a global cross-sectional displaced configuration, as Fig. 7b shows. Systems II and III also relied on an equivalent modelling with respect to linear constraints at the end sections of each girder.

For sake of symmetry, the four nodes at the flanges tips have been chosen as the "*x*-free" ones, and all other nodes were consequently the "*x*-constrained" ones. Doing so allowed fulfilling Bernoulli's "plane sections remain plane" beam theory assumption through shell models. This modelling technique has been shown to be very effective from a numerical point of view, and was validated and adopted in many F.E. studies (Greiner et al., 2009).

Modelling of *support conditions at intermediate supports* (support #2 in system II and supports #2+3 in system III) offered the following possibilities, independently:

- Vertical stiffeners in the web of the girder may or may not be considered. Consequently, web crippling at the intermediate support could be disregarded, and focus kept on the L.T.B. behaviour and response of girders;
- Lateral support possibilities at these locations included:
  - Bracing of the top flange or no bracing;
  - Bracing of the bottom flange (top of supporting column) or no bracing.

Hinges locations could include the following modelling possibilities:

- Vertical stiffeners may be added on either sides, both sides or no stiffener at all;
- Lateral bracing may be effective on top/bottom flange, on one side of the hinge, both sides of the hinge or even no lateral support at all. Indeed, according to the very limited torsional stiffness brought by the simple hinged connections usually designed, top and bottom flange bracings at the tip of the cantilever for example cannot be deemed sufficient to assume lateral bracing of the drop-in segment as well. Therefore, various combinations ought to be studied.

In order to further realistically represent the lateral support conditions of true girders, additional top flange lateral supports in-span were added so as segments of maximum 3 *m* length may remain free of any transversal support. As for System II arrangements, specific cases with top flange support/no support (free end) at the cantilever's edge were considered.

All such possibilities have been considered in the numerical parametric studies detailed hereafter, and their influence on the systems behavior and resistance shall be investigated.

Loading consisted primarily of uniformly distributed loads applied at the top flange. Variations in intensity within the various spans could however be considered, through distinct q_A, q_{B+C+D} and q_E values – see F. 4 to 6. The application of a potential additional end moment at the member's right side in systems I to replicate negative bending moment situations at the back-span extremity was also implemented.

3.4 Initial imperfections

Material imperfections have been accounted for by means of so-called "residual stresses" distributions. These "membrane" stresses are indeed known to have an influence on the carrying capacity of beams prone to an L.T.B. failure mode. In this study, 3 different types of residual stress distributions have been considered (see Fig. 8): triangular, parabolic and welded. They are defined so that the various stresses distributions are in auto-equilibrium, preferably within each plate (Gérard et al., 2019).

Geometrical initial imperfections have been defined as a combination of initial lateral deflections and initial torsional twist. Classically, the 1st global eigenmode shape has ben considered and scaled with a maximum amplitude of L / 1000, L being the average between L_A

and ($L_B + L_C + L_D$). This procedure implied that for any desired failure load reference result (G.M.N.I.A. calculation), a preliminary L.B.A. calculation was necessary.

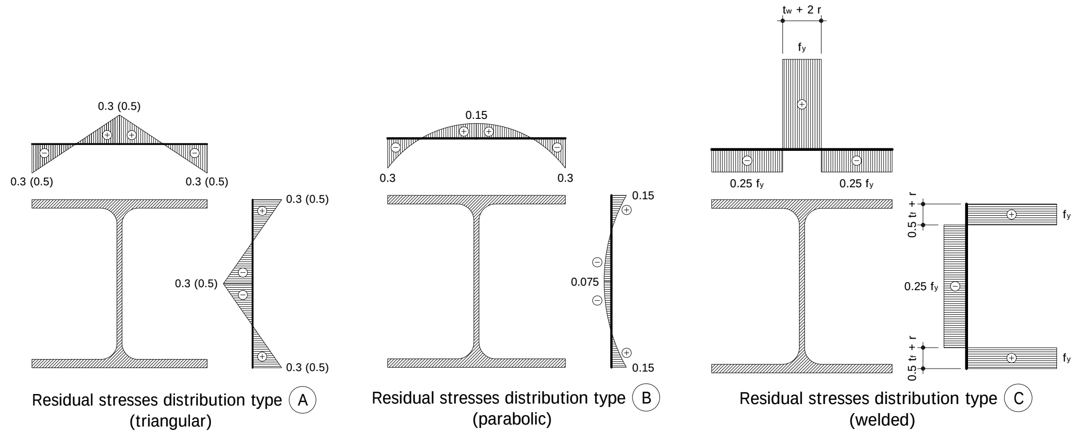


Figure 8: Possible residual stresses patterns

3.5 Validation of F.E. tools

Adequate mesh density studies have been conducted, and various situations from coarse to very dense have been considered. *Hinge behavior* validation studies have also been performed by means of elastic linear calculations (Silva et al., 2017). Maximum vertical displacements obtained with FINELg's shell models for systems I to III were compared to alternative beam elements models and correspondence between both sources of results was excellent, as not more than 6% difference could be reported for all cases, whatever the static system considered.

Then, further investigations have been performed to validate the adequacy of the shell models to provide accurate L.B.A results. In such cases, more parameters and cross-section properties (e.g. torsional inertia) play an important role. Since no dedicated software was available for Gerber arrangements, comparisons are made for system I cases with software LTBeam (2017). A maximum deviation of 5% between the two numerical sources was observed, which is remarkable owing to (i) the great sensitivity of critical load calculations to initial parameters and to (ii) the very different nature of the models compared – beam vs. shell models.

Last, results relative to system III models with different ways of modelling the hinge in the shell models were tested. Cases where the hinge consisted in joining only 10% of the two adjacent girders' height to cases where 50% of the height is connected were investigated, as well as a fully-continuous reference case where both webs and flanges were uninterrupted. Although disconnecting completely the flanges and joining only parts of the webs results in a section carrying nearly no bending moment, it does have an influence on 1st order deflections. The shell model deflections were clearly seen to be intermediate between the two extreme cases of a perfect hinge and of a fully continuous solution, when compared with beam models as well as with FINELg continuous beam solutions. Choice was finally made to keep the most detrimental configuration towards L.T.B. and to use the web-only connection between the two girders, with a $Height_{hinge}$ / $Height_{section}$ = 20% ratio.

4 F.E. parametric studies

4.1 Cases and situations considered

Lengths: the various lengths in the girder have obviously been of key influence on the L.T.B. behaviour of the system. In this respect, two different length ratios have been considered: the first set of lengths was chosen such that lengths represented twenty times the heights of the sections, while the second one considered thirty times these heights. Moreover, a ratio between the length between the two supports in the middle (i.e : the overhangs and the unsupported beam for System III ($L_B+L_C+L_D$)), L_B, L_C and L_D corresponding to the lengths shown in Fig. 6) and the first girder – common to all systems with length L_A – has be tested as 1, 1.2 and 1.4 for both lengths above. The dimensions of the girders are given in Table 2 and Table 3.

Table 2: Parameters for each section with the smallest span length

Case h x 20						
	h [mm]	L_A [m]	L_B [m]	L_C [m]	L_C [m]	L_C [m]
Sections				$\frac{L_a}{L_b+L_c+L_d}=1$	$\frac{L_a}{L_b+L_c+L_d}=1.2$	$\frac{L_a}{L_b+L_c+L_d}=1.4$
W360x33	349	6.980	1.536	3.909	2.745	1.915
W610x113	608	12.160	2.675	6.810	4.783	3.335
W840x210	846	16.920	3.722	9.475	6.655	4.641

Table 3: Parameters for each section with the greatest span length

Case h x 30						
	h [mm]	L_A [m]	L_B [m]	L_C [m]	L_C [m]	L_C [m]
Sections				$\frac{L_a}{L_b+L_c+L_d}=1$	$\frac{L_a}{L_b+L_c+L_d}=1.2$	$\frac{L_a}{L_b+L_c+L_d}=1.4$
W360x33	349	10.470	1.536	5.863	4.118	2.872
W610x113	608	18.240	2.675	10.214	7.174	5.003
W840x210	846	25.380	3.722	14.213	9.983	6.961

Support conditions and stiffeners: for all cases, a lateral support was added to the bottom flange of beams at the supports with an overhang part (i.e. one for System II, just before the overhang part and two for System III and further). Besides, for lengths of the girder superior to 3 *m*, lateral supports were added systematically on the upper flange every 3 *m* in average as explained previously, depending of the length of the girders: these lateral supports intended at representing secondary beams relying on the main girder and bringing lateral support.

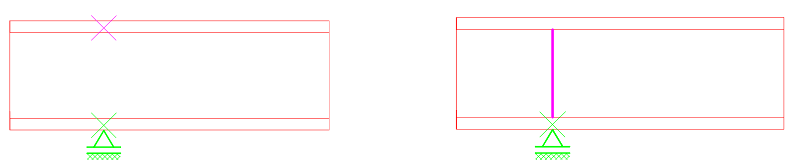


Figure 9: Overhang support – a) Lateral bracings ("BA" configuration) – b) Optional stiffener at support

For Systems II and III, possibilities to add lateral supports on the upper flanges at the same places than the two basic supports described previously were taken into account; the latter additional support permitted to get closer to fork conditions, and this case is designated as "BA" in the following. Other lateral supports could be placed at the extremity of the cantilever, on the upper and the lower flanges. Four different cases were considered, namely B0, B1, B2 and B3, see Fig. 10a and 10b; these possibilities allowed to investigate the influence of lateral supports on the resistance of the system against L.T.B. at different critical points. Various vertical stiffening configurations were tested as well, at the same places as the lateral supports. Again, intention was to study the influence they may bring to the resistance against L.T.B. All possibilities of additional lateral supports or stiffeners have been tested numerically in consecutive F.E. parametric studies, as detailed in the following.

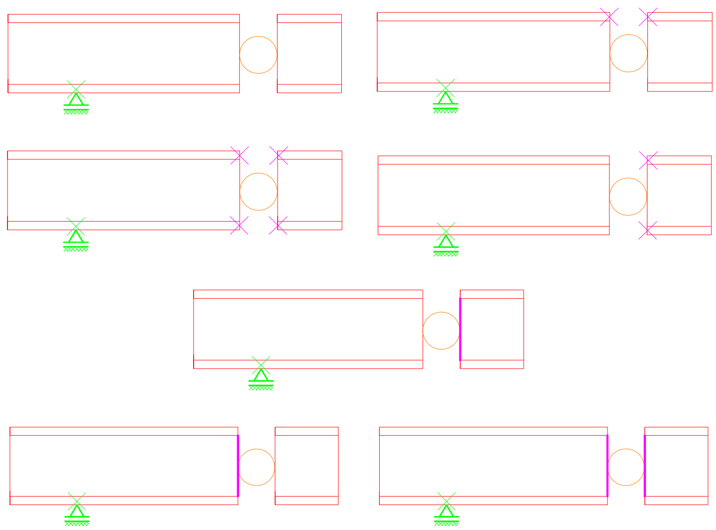


Figure 10: a) Lateral supports possibilities at hinge (Configurations B0, B1, B2 and B3) –b) Optional vertical stiffeners at hinge (no stiffener, one side only, both sides)

4.2 Results for systems I, I and II*

Results obtained for sub-systems I, I* and II (Fig. 11) are reported in this paragraph; they aim at preliminary characterizing the influence of different sub-parts of the Gerber arrangement – namely the drop-in segment, edge segment and cantilever – and at assessing S16 recommendations in such cases. Table 4 summarizes all F.E. simulations performed.

Table 4: Cases considered for Systems I, I* and II

	System I		**System I***	
	3	Different sections	3	Different sections
	2	Length L (h x 20, h x 30)	2	Lengths L (h x 20, h x 30)
	3	Length ratio (1, 1.2, 1.4)	3	Length's ratio (1, 1.2, 1.4)
Total	**18**	Cases	**18**	Cases
	System II			
	3	Different sections		
	2	Lengths L (h x 20, h x 30)		
	3	Length's ratio (1, 1.2, 1.4)		
	2	Cases of stiffeners on support		
	2	Cases of stiffeners at the hinge		
	2	Cases of lateral supports on support		
	3	Cases of lateral supports at the hinge		
Total	**432**	Cases		

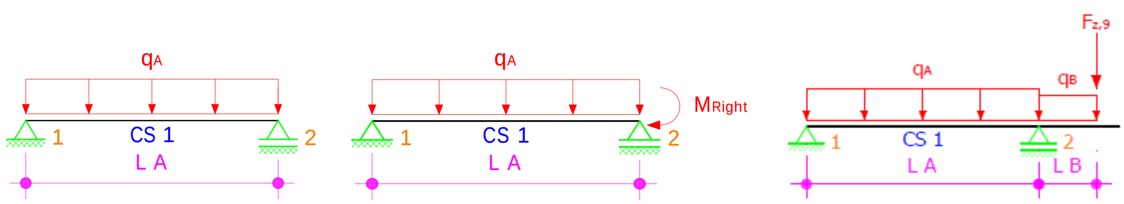


Figure 11: Evaluated systems – a) System I – b) System I* –c) System II

4.2.1 Results for System I

System I basically served as a basis to compare the F.E. results with resistance predictions from the Canadian Standards and the Beaulieu et al. proposal (Eurocode 3 (2005) predictions are also sometimes reported on the figures, for sake of completeness). By means of an L.B.A. analysis (Silva et al., 2018), the analytically-predicted critical bending moments could be compared with the numerical reference ones (see Fig. 12a). The observed discrepancies are globally quite important:

- First, S16 is seen to underestimate by as much as 50% the system's capacity, in large extents owing to its special rules for transverse loads applied on the upper flange that requires the length to be arbitrarily to be augmented by at least 20%;
- Then, the effect of placing a lateral support on the upper flange every 3 *m*, assumed as being sufficient for setting a fork support according to the standard, can be shown incorrect and responsible for some the unsafe results presented in Fig. 12a.

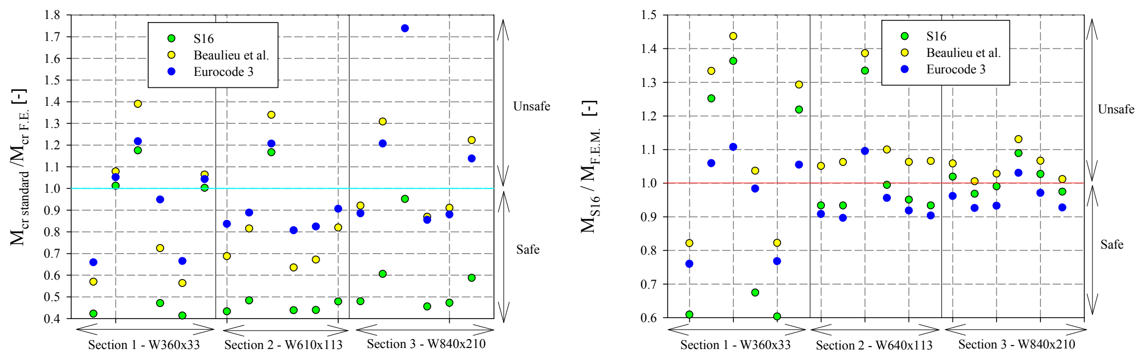


Figure 12: F.E. vs. analytically-predicted bending moments – a) Critical (L.B.A.) – b) Ultimate (G.M.N.I.A.)

In the same way as for L.B.A. simulations, G.M.N.I.A. results were compared to S16 resistance predictions, as displayed on Fig. 12b, which bring the following information:

- Differences are more important for the first section (W360x33), similarly to results presented in Fig. 12a;
- Whereas results for LBA cases sometimes presented large discrepancies between code and FE results, their GMNIA counterparts are seen quite more consistent – usually 10% deviation in average;
- Detailed analysis of Section 1 results allowed to evidence that the combination of a small height with extra lateral supports can result in a significantly more stable girder, thus the important resistance reserves reported in Fig. 12b for Section 1.

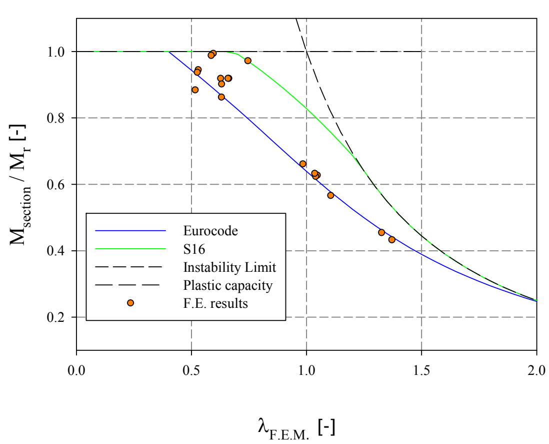
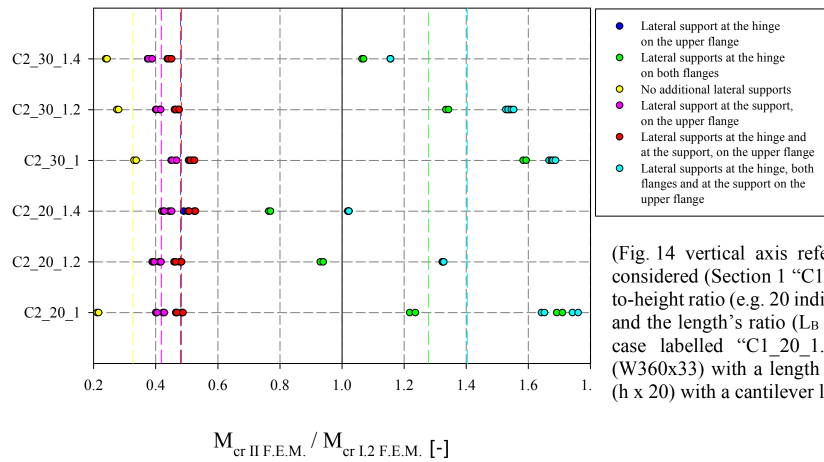


Figure 13: Graphical comparison of F.E. results and S16 resistance curve

Besides, typical relative resistance – slenderness λ plots were prepared, such as the one presented on Fig. 13, where refined values of slenderness $\lambda_{F.E.M.}$ were determined from the more accurate FINELg's critical moments. The intention was to assess the ability of S16 to correctly estimate the carrying capacity of the girders once the critical moment is made sure to be accurate. As Fig. 13 results clearly shows, S16 usually fails predicting safe resistances and leads to inaccurate results overestimating the capacity of the girder (by some 20% at most), where, in contrast, other design rules – Eurocode 3 rules in the present case – may perform better. This further suggests a need for improving S16 L.T.B. equations, in addition to adding recommendations relative to the buckling behaviour of Gerber systems.

4.2.2 Results for System I and System II*

System I* was intended at updating the bending moment diagram with a sagging part at the right end (see Fig. 11b), the girder remaining on ideal fork support conditions. System II aimed at determining the influence of the cantilever part on the system, and therefore could account for failures possibly driven by the cantilever part, as well as consider the complex interactions between the back-span and cantilever segments. Both systems shared the particularity to be submitted to both sagging and hogging bending moments, so that the girder may buckle by either excessive compression on the upper flange or by L.T.B. in the lower flange.



(Fig. 14 vertical axis refers to, respectively, the section considered (Section 1 "C1" to Section 3 "C3"), the length-to-height ratio (e.g. 20 indicates a beam for which L = 20 h) and the length's ratio (L_B = 1.0 to 1.4 L_A). For example, a case labelled "C1_20_1.2" refers to the first section (W360x33) with a length of 20 times the section's height (h x 20) with a cantilever length L_B = 1.2 L_A)

Figure 14: Comparison of the critical moment between Systems II and I* for section 2

The corresponding results are reported on Fig. 14, which yields the following comments:

- Conditions of lateral support have a considerable influence on the values of the critical moments. Modifications in lateral bracing may even lead to a change in the first eigenmode;
- If no lateral support at the extremity of the overhang is present, then L.T.B. becomes determinant;
- Supporting both upper and lower flanges leads almost always to an increase in critical moment.

As differences between the values of critical moments between the two systems are seen to be so important, it is concluded that relying on System I* only is not appropriate. In other words, the influence of the cantilever on the back-span stability, as very much influenced by flange bracing, is so significant that one shall refer to System II and its many variants. The results of L.B.A. simulations were also compared with the results from various standards' equations, and an example is plotted in Fig. 15. The Canadian code still is seen to both over and underestimate the capacities of the girders, in large extents. The large discrepancies noted previously get even worse here, as S16 poorly addresses the influence of lateral supports on critical moments.

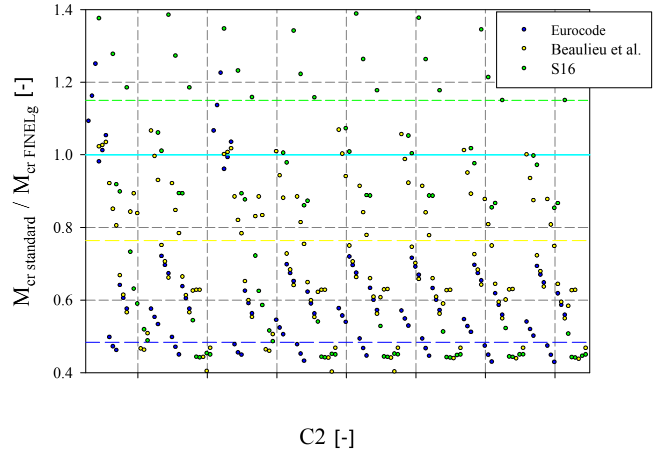


Figure 15: Comparison of System II critical moments for section W610x113

Results for G.M.N.I.A. investigations are presented in Fig. 16, from which the interesting observations can be summarized as follows:

- Even if the critical moment was seen increasing through improved bracings, resistance may be unaffected – this is typical for relative shorter spans, as governed by resistance;
- The presence of supports on the overhang usually leads to similarities in behavior and resistance with System I*;
- These behaviors sometimes may appear somewhat "random" depending of the supports condition, section and length. This can be explained by the fact the first eigenmode changes from case to case, leading to different failure modes (i.e. at different locations), which makes an analogy more complicated and delicate.

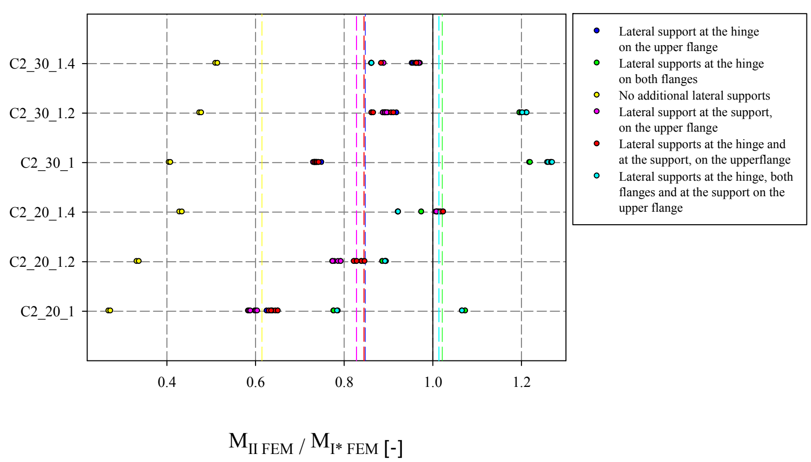


Figure 16: Comparison between Systems I* and II ultimate moments for Section 2 (W610x113)

Fig. 17 further compares F.E. results with various code predictions; here again, CSA S16 is seen rather inappropriate at providing safe and accurate design resistances for System II.

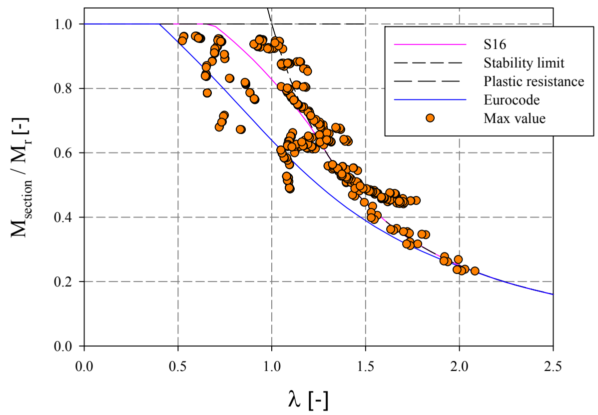


Figure 17: Graphical comparison of F.E. results for System II and S16 resistance curve

4.3 Results for system III

Investigations relative to System III included more variations in stiffening and lateral support possibilities at hinges (details in Table 5), as these systems comprised girders on both sides of hinges so that bracing could be added on the left side or on the right side (or both) of hinges.

Table 5: Cases considered for System III

		System III
	3	Different sections
	2	Lengths L (h x 20, h x 30)
	3	Length's ratio (1, 1.2, 1.4)
	2	Cases of stiffeners on supports
	4	Cases of stiffeners at hinges
	2	Cases of lateral supports on supports
	4	Cases of lateral supports at hinges
Total	1152	Cases

4.3.1 L.B.A. results

A detailed study of the 1st eigenshape first allowed to eliminate several cases where a lack of lateral bracing lead to extremely low resistances. Adding lateral supports to bring stability in the vicinity of the hinges was indeed essential: a system with no lateral brace at the hinges was shown to possess such a low lateral rigidity that it was simply unable to provide any stability to the central drop-in segment.

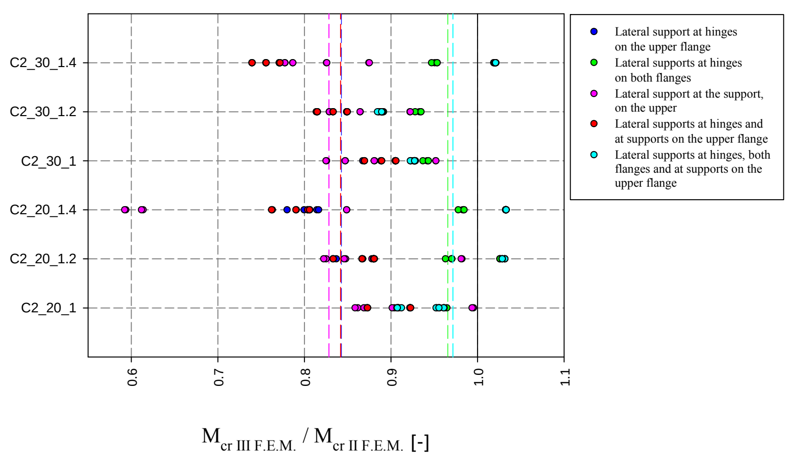


Figure 18: Comparison of critical moments for Systems II and III – Section C2 (W610x113)

Fig. 18 generally evidences that the critical moments calculated in System II were overall superior to those in System III, except in the most laterally supported cases. This was expected as a consequence of less favourable global support conditions in the full system, as compared to less “segments” in System II.

To isolate the influence of the length on the overall trends of results, Fig. 19 plots the evolution of ratio of the “current” critical moment to the critical moment of a reference case with no stiffeners and no lateral supports, the “current” situation being progressively stiffened and with more bracings.

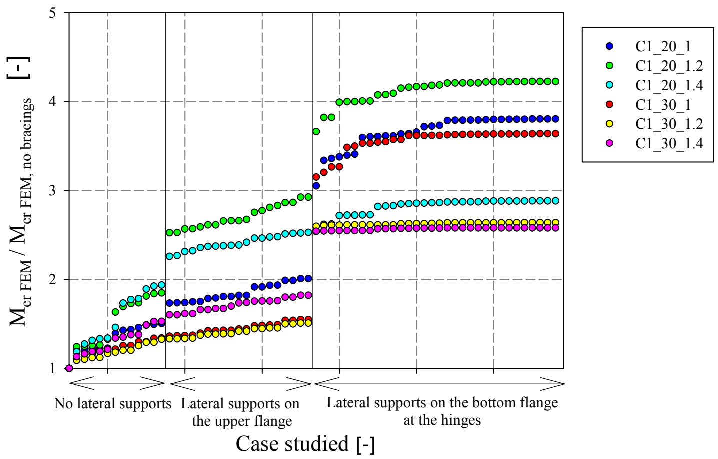


Figure 19: Influence of lateral supports on the critical moment – First section (W360x33)

Analysis of Fig. 19 brings the following main information for Section 1 (W360x33):

- The first part of the figure (left, no lateral supports) is characterized by low M_{cr} values; however, adding vertical stiffeners may have an interesting influence, depending on the girders’ slenderness – see increases in critical moment up to some 50%;
- The second part evidences that placing lateral supports on the upper flange appreciably increases the buckling resistance, particularly for the girders with a little slenderness. The position of stiffeners also is seen to influence the resistance;
- The third part demonstrates that a maximum resistance is quickly attained, no matter the support on the bottom flange brought in addition. Use of stiffeners however help increasing the resistance, depending of their positioning;
- For larger beam slenderness (L= 30 x h) and smaller relative length of overhang segments (1.0), the gain is usually higher;
- The greater the girder’s slenderness, the lesser the (positive) influence of stiffeners (last two zones).

Similar tendencies were observed for all sections. These analyses evidence that the various lateral support conditions considered have a key influence on the resistance of the system. As a result, the case with the stiffeners on both sides of the hinge coupled with top and bottom flange lateral supports was shown to always be the best solution since, in such a configuration, the girder behaves like a continuous girder.

4.3.2 G.M.N.I.A. results

Results reported on Fig. 20 for G.M.N.I.A. analyses first show that the more slender beams with large span ratios are more subjected to an important increase in resistance when support conditions improve – the C2_20_1.4 case increases significantly only for Section 2. Resistance is indeed seen to drastically increase as soon as the lateral supports are added. Situations associated with the higher impact on resistance are seen to consist in adding lateral supports and stiffeners at the hinges locations.

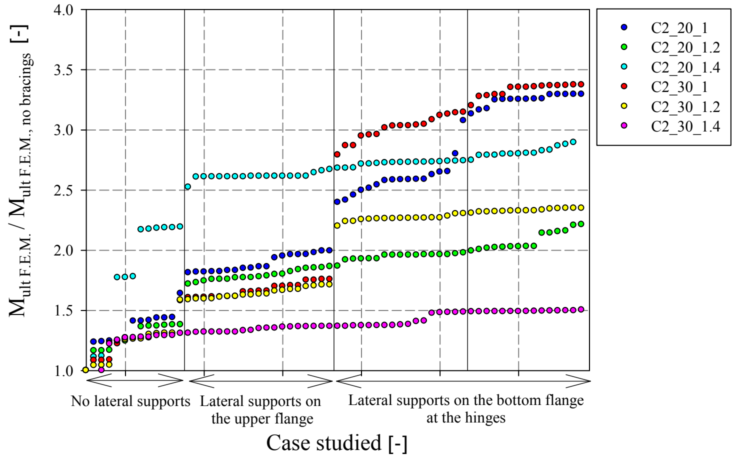


Figure 20: Comparison of the ultimate resistance for the second section (W610x113)

4.4 Influence of drop-in segment section change

As a great possibility offered by the Gerber system and frequently used in practice, changing the drop-in segment section for a deeper or a smaller size is deeper investigated in the present paragraph. Because the first series of results relative to System III pointed out that some support configurations should not be investigated any further, only six different configurations were finally considered: two dispositions of lateral supports at the hinge as represented in Fig. 21a and 12b and three dispositions of stiffeners as represented in Fig. 21c to 21e; the corresponding cases investigated are summarized in Table 6.

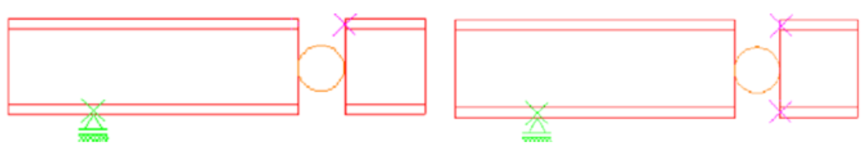


Figure 21: a) and b) Possibilities for lateral supports – c), d) and e) Possibilities for vertical stiffeners

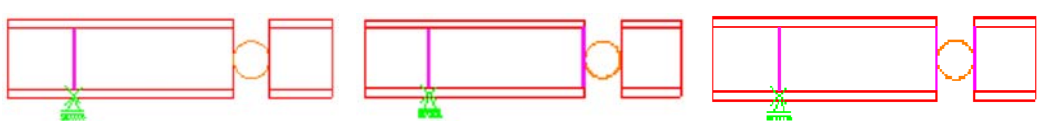


Table 6: Cases considered for System III and drop-in segment section change

Remaining systems		
	3	Different sections
	2	Lengths L (h x 20, h x 30)
	3	Length's ratio (1, 1.2, 1.4)
	1	Case of stiffeners on support
	3	Cases of stiffeners at the hinge
	1	Case of lateral supports on support
	2	Cases of lateral supports at the hinge
Total	108	Cases

Fig. 22 plots the obtained results, both for L.B.A. and G.M.N.I.A. calculations, and provides the following information:

- Even if a change in section for the simply supported part (CS2) slightly undermine the critical moment, the latter does not change significantly, so that the economic gain is really important in comparison to the loss in stability;
- The reduction of CS2 seems to affect more the system's resistance for the new bigger sections (W840x210) compared to previous sections W360x33 and W610x113;
- Similar trends for less laterally supported cases are noticeable but for more supported cases, the critical moment is inhibited a lot in comparison to unique section cases. These significant reductions in critical moments seem linked to a more important difference of height between CS1/CS3 (back-span and overhang sections) and CS2 (drop-in segment section). It may be explained by the use of a smaller hinge, which possibly affects the system's resistance and stability;
- Although collapse often remained on the bigger cross-section (CS1), the way the hinge was made effective – joint details – triggered additional restraints from the drop-in segment (CS2). Accordingly, a smaller hinge height associated to a reduced CS2 may explain the decrease in resistance observed for the section changes. Particularly, differences in section's height, hinge's location and size between CS1 and CS2 are determinant. The closer heights, the more resistance in the system;
- Well-suited support conditions can also bring significant resistance to such system as the resistance can be significantly increased (even tripled, see case C3_20_1 for example). Accordingly, they shall be considered as a way to reduce the amount of steel in Gerber systems very susceptible to L.T.B.

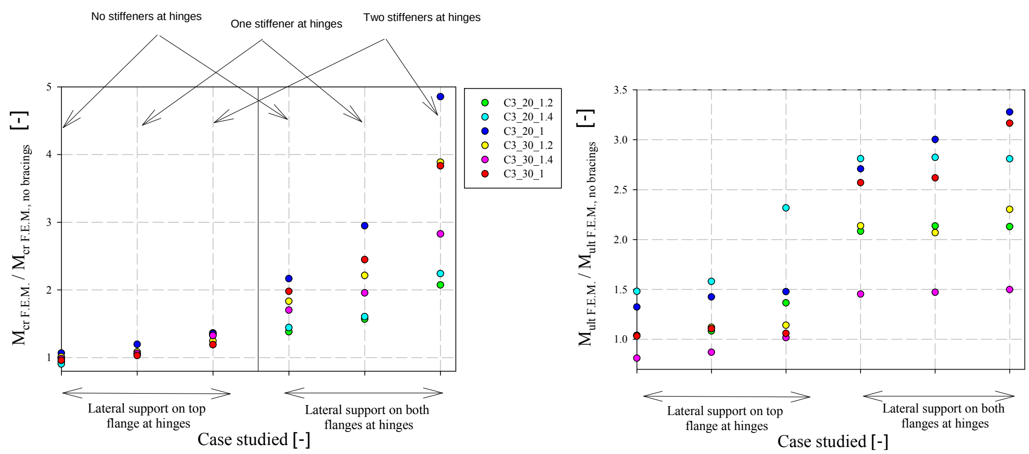


Figure 22: a) Influence of support conditions on critical moment – Section 3 (W840x210) – b) Influence of lateral supports on resistance – Section 2 (W610x113)

4.5 Uplift loading

In this section, the same system is studied but the load is reversed. i.e. applied vertically upwards. Such cases usually exhibit a lower load level but also a lower resistance, since support conditions are optimized for a downward load. This may lead to a relative weakness that shall be further studied. Place is missing here for being comprehensive, and one may refer to (Manaud et al., 2018) for more details. L.B.A results for the first section (W360x33) are summarised in Fig. 23a. For identical lengths, no major changes in critical moments are observed, whatever the support conditions considered. This appears logical since lateral supports are firstly meant at bringing stability on the part of beams in compression under vertical *downward* load, which is not the case here. Consequently, much longer spans than in § 4.4 are contemplated in the following, as a result of no lateral supports on bottom flanges.

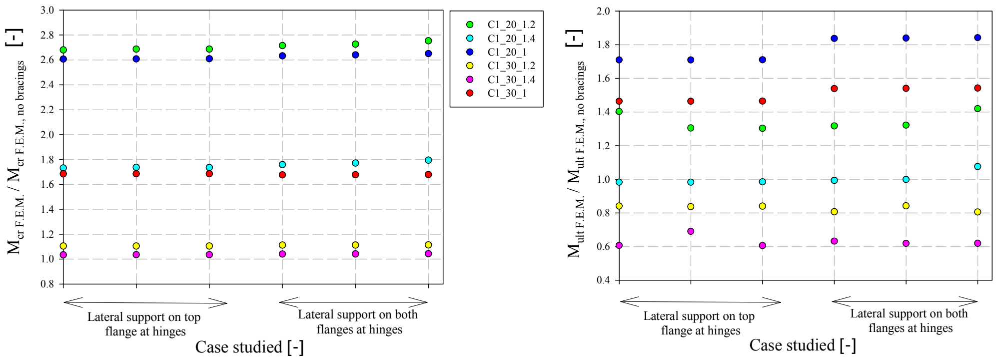


Figure 23: Results for negative load and Section 1 (W360x33) – a) Critical moment – b) Resistance

These analyses obviously evidence the inability to provide sufficient lateral support to the compression zones in case of uplift loading. Most of the cases however have a critical moment ratio superior to 1.0, which means that some support conditions provide more stability, such as stiffeners at columns or lateral supports around columns for example. They also permit to see the ability of CS2 to affect the system's stability. In particular, the difference in height between CS1 and CS2, and therefore the size of the hinge, is a parameter that shall be carefully considered during the design.

G.M.N.I.A. results for Section 1 (W360x33) are summarised in Fig. 23b.As for G.M.N.I.A results, different trends can be observed, usually depending on span length, but globally an identical resistance is reached for an identical length. The inability of the support conditions to provide additional system resistance to an uplift loading is highlighted again – this, in some cases, may not be a crucial issue since this limited resistance shall be compared to a relatively lower load intensity. Detailed analysis of Section 2 (W610x113) results showed that unlike for Section 1, some lengths tended to be sensitive to support conditions, particularly cases with stronger bracing conditions at the hinge. The cantilever part was indeed seen to buckle together with the back-span part for cases without stiffeners, while only the back-span part buckled in cases with two stiffeners: in the latter, the more laterally-supported hinge helps the drop-in segment bringing more restraint. Results for section 3 (W840x210) are similar to those of section 2. As for Section 3 and as expected, slender systems seemed more susceptible to support conditions than less slender systems. The height of drop-in sections was shown a key parameter, and as soon as the resistance is strongly associated to the critical bending moment, an accurate and reliable determination of M_{cr} is here again crucial.

4.6 Fully-braced top flanges

These extra simulations aim at observing the behaviour of Gerber systems where beams are assumed to be totally braced laterally on their top flange. This corresponds to situations where the roof deck is very rigid in its plane so that a strong diaphragm effect takes place. Obviously, such configurations usually provide better support conditions against L.T.B. than previously-studied ones. Yet, lateral stability issues cannot be assumed as totally prevented since hogging bending moment regions in the vicinity of the column are still present, where a top flange bracing only may be insufficient.

In the following figure, $M_{cr\ F.E.M.\ no\ bracings}$ refers to cases with minimum stiffening, where no lateral braces nor vertical stiffeners are present at the hinges. Top flange lateral supports every ≈3 *m* along the whole system are however present, as well as a bottom flange bracing at interior supports but no top flange nor vertical stiffeners in these sections.

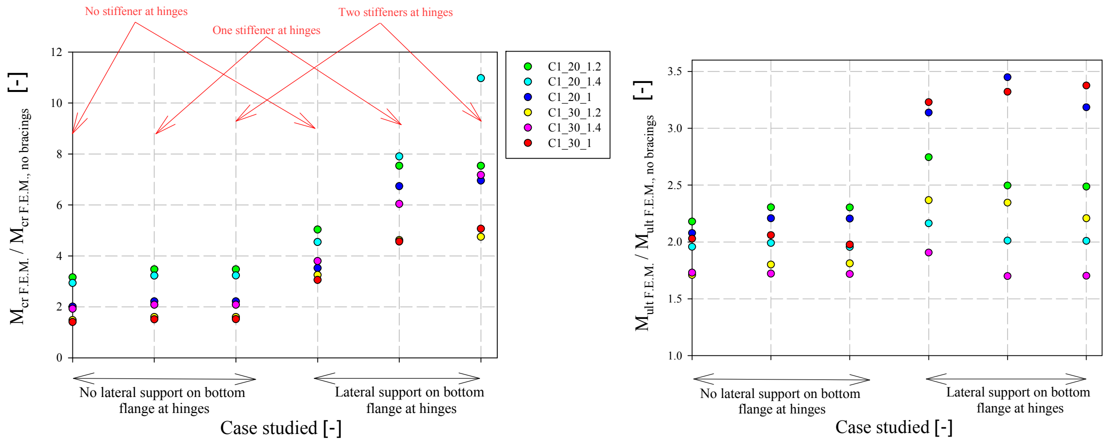


Figure 24: Results for Section 1 (W360x33) with fully braced top flanges – a) Critical moment ratios – b) Resistance ratios

Fig. 24 report the obtained results for Section 1, for both L.B.A. and G.M.N.I.A. analyses (results for Section 2 and Section 3 yield similar comments):

- For realistically slender systems, laterally bracing the top flange usually significantly increases system's overall stability. This strongly suggests that benefiting from a diaphragm effect from the roof panels may lead to more economical structural solutions;
- Occurrence of a detrimental lateral instability however remains in cases where the length of the hogging moment region is important. In the latter cases, the benefits of continuously supported top flanges is obviously very limited;
- When no bottom flange lateral supports at hinges are present, some instability modes consist in the bottom fibres of the section to laterally buckle and in the section to rotate about a longitudinal axis located at the top flange level (Fig. 25). Vertical stiffeners at the hinges shall only result in the drop-in segment to rigidly rotate without buckling;
- Consequently, provided a decent level of lateral bracing (either top or bottom flanges or both) is present on the system, detailing of the hinge connection with respect to its torsional stiffness is crucial. Accordingly, the design of the hinge connection shall not be restricted to the transfer of shear forces and the different connection detail possibilities (e.g. splice connections or web end plates) are not equivalent.

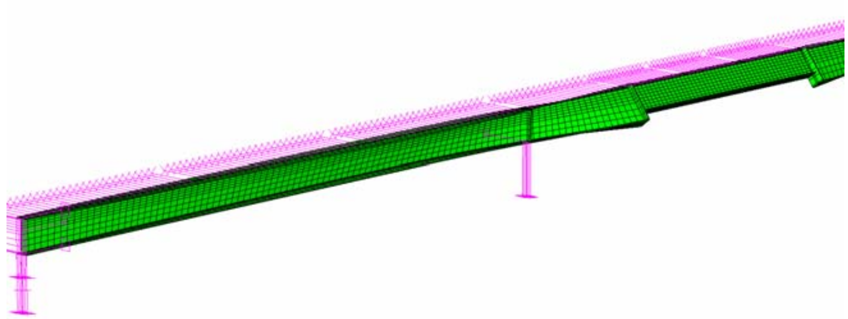


Figure 25: First eigenmode of a C1_20_1.2 case without lateral support on bottom flange at hinges

Detailed analyses and comparison of cases with full top flange bracing to cases without fully braced top flanges pointed out that some situations are seen to reach a maximum resistance when not continuously braced on top flanges, possibly indicating that L.T.B. caused by hogging moment and a lack of horizontal supports on bottom flanges governs. Another typical configuration in which the benefits of continuously bracing the top flanges has virtually no influence corresponds to cases where cross-section plastic resistance prevails: in such situations, lateral stability is made sufficient through discrete bracings, so that increasing it by means of bracing all top flanges has no real effect on the system's overall resistance.

As a conclusion, one may keep that designing the roof deck so as to benefit from an effective diaphragm effect has a positive influence of the system's overall resistance. In such cases, the top flanges may be assumed as fully braced laterally and the system's resistance becomes governed by its resistance to L.T.B. in hogging moment zones. Provided lateral buckling of the bottom flange in hogging moment regions is reasonably controlled, resistances close to the plastic capacity may be reached. Further studies (Manaud et al., 2018) allowed to conclude that, for fully braced top flanges configurations, the presence of bottom flange lateral bracings at the hinge positively affects the system's overall resistance, sometimes in great extents. Under such conditions, the careful detailing of the hinge connections is key to improving further the performance, namely with respect to transferring torsional forces between adjacent members – see next paragraph. In this respect, the presence of vertical stiffeners at end sections near the hinge are seen a plus when a splice connection is designed.

4.7 Influence of joint configuration

An ideal hinge connection shall be able to transfer the torsional forces triggered by the torsional component of a potential L.T.B. failure mode. In this respect, typical splice connections may appear quite weak unless they are complemented by vertical stiffeners in their vicinity, as previously studied – this however remains unpractical, as well as more expensive, simple and cheap hinge connections remaining key to the system's efficiency and popularity.

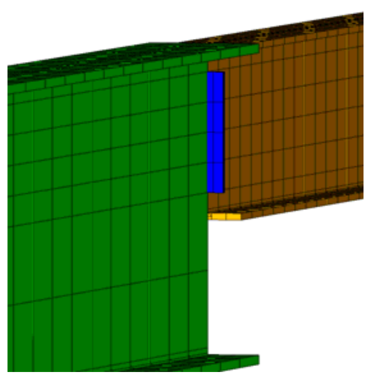


Figure 26: Model of web-end plate connection

As an alternative economical yet easy hinged connection, use of web-end plates (Fig. 26) may allow to fulfil the hinge requirements while improving the joint's torsional stiffness. The latter type of connection was studied numerically through a set of two end plates per hinge, assumed as welded along a fraction of adjacent girders' web heights. Cases with so-called "small" end-plates (i.e. 20% of beam's depth, average if the two sections have different heights) as well as "high" end plates (i.e. 80% of the height) have been considered. They are connected by means of two bolt rows (modelled as cylinders), which number varies with the number of nodes in the end-plates. Fixed diameter of 20 *mm* and strict elastic behaviour of the bolts were considered in the numerical models, aiming at eliminating (local) joint failure.

Fig. 27a and 27b provides examples of the obtained results, from which the following observations could be drawn:

- Use of small end plates does not increase the system's overall stability, as important local deformations in the girders' webs get often captured as the 1st eigenmode. Relying on increased web thickness and height usually are sufficient to increase the system's critical load;
- Some configurations can be seen as "equivalent". For the C2_20_1 cases for example, the response of the system with small end-plates is similar in load and 1st eigenmode to that of the splice with a single vertical stiffener case;
- Large end plates obviously exhibit the best results and are seen to lead to load ratios as high as for the double stiffener cases. They provide sufficient torsional stiffness to the joint, reasonably prevent local buckling while remaining simple and economic. Therefore, adopting high web end-plates connections as hinges appears to be the best choice for an optimal system's performance – as a reminder, the height of a "high" end-plate was set to 80% of the height of the assembled girders;
- Quite unexpectedly and unlike for L.B.A. calculations, ultimate resistances of girders with web end-plate hinges are generally lower than their spliced connection counterparts. This shall in large extents be explained by a specific local plastic collapse mechanism in the girders' webs. Such lower failure load levels are usually not captured in splice models since the concentration of strains mostly affects the plate elements representing the splices which were set to remain elastic. In contrast, small web end-plates lead to high local stresses and ultimately, plastic failures in the webs. Therefore, comparisons between models should be made with great care. For large end-plates situations, smoother distributions of stresses lead to much higher load ratios, usually comparable to cases with splices with two stiffeners at the hinges;
- Even if large end-plates cases lead to a local collapse mechanism, their strength remain close to cases with splice connections. Similar global failure modes are observed, further indicating a certain equivalence between models;
- Some cases appear little affected by joint modelling, as a result of quite slender girders failing early from lack of lateral stability – thus with little dependence on the joints configurations.

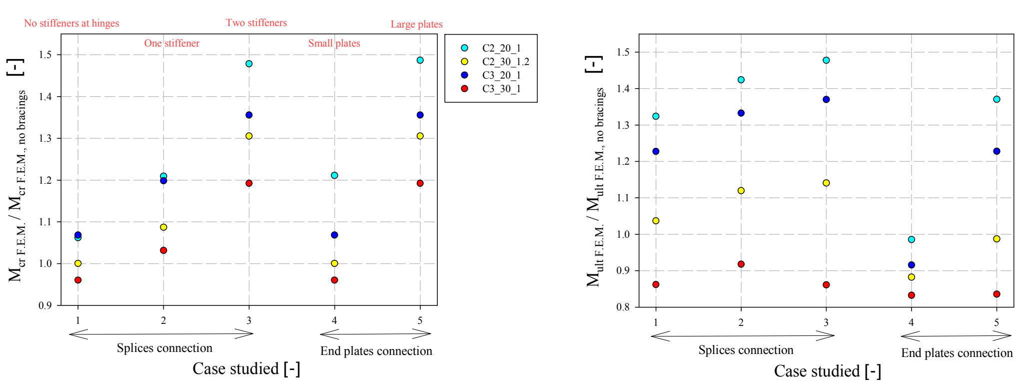


Figure 27: Influence of joint modelling – a) On critical moment ratios – b) On resistance moment ratios

Overall, the numerical results indicated that the use of large web end-plates can be deemed as rather similar in behaviour and strength to hinges made with splice connections with two vertical stiffeners, however presumably at quite lower costs. In particular, web end-plates were seen to quite correctly transfer the torsional forces arising from L.T.B. global failure modes.

5 Design proposal and practical recommendations

5.1 Practical design recommendations

The following provides a summary of practical guidelines that should be followed for an optimum performance of a Gerber system towards L.T.B., based on the present paper observations:

- Top flange lateral bracing at each secondary beam (i.e. loading point) shall be effective;
- Vertical stiffeners and bottom flange lateral bracing must be placed at sections of internal supports (at sections above internal column);
- Hinge locations should ideally be braced laterally on both top and bottom flanges, preferably on the drop-in segment or on the smallest section;
- At hinges, web end-plate connections with suggested at least 80% of the height of the smallest section shall be preferred;
- Continuous lateral bracing of the top flange through adequate diaphragm effect, if possible, is significantly beneficial.

5.2 Design proposal

Section 4 allowed to point out that the current S16 design rules for L.T.B. are inappropriate to provide accurate and safe resistance predictions. One of the most influential parameter was shown to be the critical moment, whose calculation proved quite delicate and challenging, owing to many possible bracing and stiffening configurations leading to substantial changes in M_{cr}. Besides, the remaining of the design equations as of CSA S16 were also put into question in the light of the numerical results collected.

In this respect, the following paragraphs investigate in which extent a design approach based on accurate M_{cr} values, calculated by means of appropriate tools (e.g. beam finite element models), and associated to S16 resistance equations can be proved satisfactory. Alternatively, other design equations – Eurocode 3 ones here – may be considered and associated to this more accurate critical bending moment.

5.2.1 Performance for system III with and without changes in drop-in segment section

The suggested design approach was confronted to the numerical results detailed along § 4. A first set of comparison data is presented in Fig. 28, which reports the cumulative frequencies of the reference-to-predicted bending resistance ratios from current S16, proposed approach (F.E.-calculated M_{cr}) with S16 and proposed approach with Eurocode 3 (EC) buckling curves.

Firstly, the current S16 provisions are seen inaccurate, both vastly unsafe and over-conservative, since large proportions of the corresponding results belong to the extreme parts of the graphs, which are the worst. Although safe-sided in average, strict S16 predictions also show a lot of scatter, denoting their inadequacy. In contrast, S16 coupled with accurate determinations of M_{cr} show more consistent results; however, a lot of results are seen to lie on the highest ranges of unsafety, i.e. ratios above 1.3, indicating that the sole replacement of the critical moment is not sufficient to regain safety and consistency. This particularly holds true for Fig. 28b data, which clearly suggest the replacement of S16 L.T.B. rules for more conservative resistance estimates. Accordingly, this design approach is advised to be disregarded or used with great care.

Finally, results based on the more conservative buckling curves of Eurocode 3, coupled with an accurate value of M_{cr}, show to be the best resistance predictions: coherent and globally safe results are reported, for both situations of Fig. 28. Although some unsafe results can still be observed, they represent a limited number of situations and remain within reach of usual safety factors that shall make the final design calculations back to the safe side.

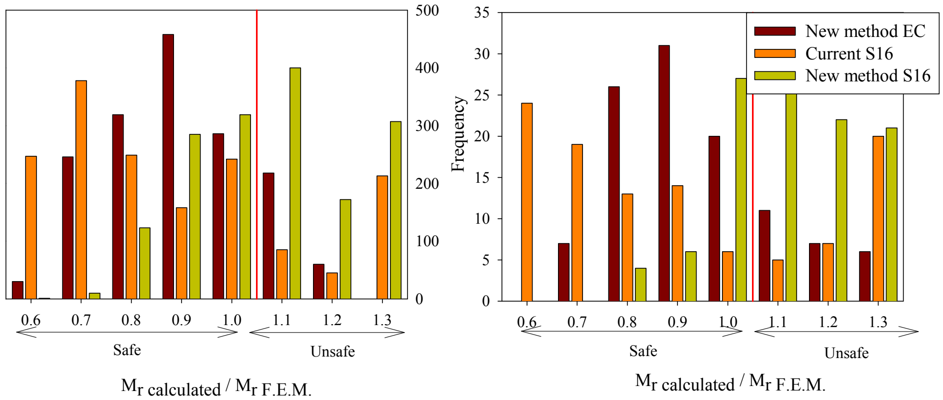


Figure 28: Statistical distribution of results for various resistance prediction methods – a) System III with identical sections – b) System III with changes in drop-in segment section

5.5.2 Performance for System III and uplift loading – Fully-braced top flanges

Similar analyses can be conducted for other cases, namely uplift loading cases or systems with fully-braced top flanges; the corresponding results are provided in Figs. 29a and 29b. Uplift loading results are seen very consistent, for all code approaches tested, owing to usual large unsupported lengths which make L.T.B. the nearly exclusive failure mode. Current S16 design recommendations lead to very conservative resistance predictions. Oppositely, the new proposal coupled with S16 remaining L.T.B. rules provides results exclusively on the unsafe side that cannot be accepted neither. Yet, the proposed approach yields very consistent and safe-sided predictions.

As for systems with fully braced top flanges, the following trends shall be reported:

- Use of the current S16 specifications now leads to very unsafe results, with more than 80% of its predictions on the unsafe side;
- The proposed new approach – accurate M_{cr} – with S16 rules still leads to highly unsafe results, however in lesser extents as nearly 50% of the results now lie on the safe side;
- Resistances obtained with the proposed approach again show a much better performance, proposing safe and consistent resistance estimates, as indicated by the low scatter in Fig. 29b.

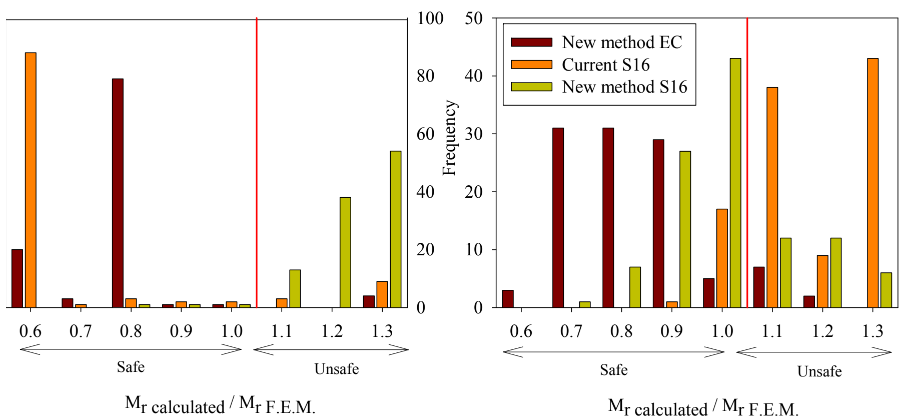


Figure 29: Statistical distribution of results for various resistance prediction methods – a) System III for uplift loading – b) System III with fully-braced top flanges

6 Conclusions

This paper presented a series of numerical results and the associated design recommendations for so-called “Gerber systems”. Based on the results of carefully-developed F.E. models, various key parameters could be identified and studied, such as system lengths, section dimensions, changes in section for the drop-in segment, uplift loading cases, fully-braced top flanges, or several joint configurations.

Various practical design recommendations could be summarised for an optimum performance of Gerber systems in practical applications. Also, proposal was made, given the complexity and the many possibilities to restrain lateral buckling, to rely on an accurately-calculated value of the critical moment (e.g. by means of beam finite element models) in association with so-called buckling curves. Although the Canadian Standards CSA S16 were shown still inappropriate, this proposal, coupled with the more conservative L.T.B. equations of Eurocode 3, showed reasonably accurate, safe and consistent.

7 Acknowledgements

The Canadian Institute of Steel Construction, by means of grant # CISC-2017-02 for project “Design of beams with overhanging segments against Lateral Torsional Buckling”, is gratefully acknowledged.

8 References

Beaulieu, D., Picard, A., Tremblay, R., Grondin G., Massicotte, B. (2003). “Calcul des charpentes d’acier – Tome 1”, CISC-ICCA, Willowdale.

CSA S16-14. (2014). “Design of Steel Structures”, Canadian Standards Association.

European Committee for Standardisation. EN 1993-1-1: Eurocode 3 – Design of steel structures. Part 1.1: General rules for buildings. CEN Brussels, 2005.

Gérard, L., Li, L., Kettler, M., Boissonnade, N. (2019). “Recommendations on the definition of material imperfections for the stability and resistance of I and H sections”, submitted to the Journal of Constructional Steel Research.

Greiner, R., Kettler, M., Lechner, A., Freytag, B., Linder, J., Jaspart, J.-P., Boissonnade, N., Bortolotti, E., Weynand, K., Ziller, C., Oerder, R. (2009). “SEMI-COMP: Plastic member capacity of semi-compact steel sections – A more economic design”, Research Fund for Coal and Steel, European Commission, ISBN 978-92-79-11113-6, 139 pages.

Hirt, M. A., Crisinel, M. (2016). “Charpentes métalliques (TGC volume 11)”, Presses Polytechniques et Uuniversitaires Romandes.

https://failures.wikispaces.com/Save-On-Foods,+Station+Square,+Burnaby,+BC (2017).

LTBeam software, v1.0.11. (2017). Centre Technique Industriel de la Construction Métallique.

Non-linear finite element analysis program “FINELg” (2012), User’s Manual, version 9.3, ArGEnCo Department, University of Liège, Greisch Info S.A., Liège, Belgium.

Manaud, V., Silva, K., Boissonnade, N. (2018) “Design of beams with overhanging segments against Lateral Torsional Buckling”, Project CISC-2017-02, Report #3.

Manaud, V., Silva, K., Boissonnade, N. (2018) “Design of beams with overhanging segments against Lateral Torsional Buckling”, Project CISC-2017-02, Report #4.

Silva, K., Manaud, V., Boissonnade, N. (2017) “Design of beams with overhanging segments against Lateral Torsional Buckling”, Project CISC-2017-02, Report #1.

Silva, K., Manaud, V., Boissonnade, N. (2018) “Design of beams with overhanging segments against Lateral Torsional Buckling”, Project CISC-2017-02, Report #2.

Yun, X., Gardner, L. (2017). “Stress-strain curves for hot-rolled steels”, Journal of Constructional Steel Research, Vol. 133, pp 36-46.