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Imperfection insensitive thin steel tubular shells under bending

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Abstract
Thin cylindrical shells are highly sensitive to imperfections, and the presence of even small geometric imperfections reduces their capacity significantly. This high sensitivity reduces the inherited benefits of thin cylindrical shells and has long been an obstacle for the effective and efficient use of thin cylindrical shells. The current practices, to deal with this imperfection sensitivity, are through the use of knockdown factors and stiffeners. These methods, which are also used for designing wind turbine towers among many other structures, diminish the structural benefits and increase the cost of construction. Recently, a new approach has emerged to reduce the sensitivity of thin cylindrical shells to imperfections, and to increase the load carrying capacity. In this approach, wavy cross-sectional shapes are explored instead of circular cross-sections. The wavy cross-section shapes reduce the slenderness (R/t) of the cylindrical shells because the local radius of curvature is reduced and consequently, the imperfection sensitivity of thin cylindrical shells is also reduced. Past studies have been carried out using the wavy shape cross-sections and they present highly promising results. These studies did not investigate the effect of residual stresses, which is the essential part of wavy cross-sectional cylindrical shells. Furthermore, an application of wavy cross-sectional thin cylindrical shells in tall wind turbine towers is explored to illustrate the benefits of wavy cylinders.

1. Introduction

Thin cylindrical shells are optimal structures (high load carrying capacity with low material volume), and they are widely used due to their structural efficiency and appeal to aesthetic. But the sensitivity of thin cylindrical shells to imperfections diminishes the structural efficiency associated with them. The presence of imperfections reduces the load carrying capacity of thin shells significantly, and imperfections are unavoidable. Thus, thin shells are designed very conservatively to account the presence of imperfections. To design thin cylindrical shells, we find the capacity of perfect shells and reduce them by a factor, known as knock-down factor, and the resulting value is assigned as the design capacity of the shells (NASA 1965). This is the accepted method for designing thin shells, and almost all the design codes, e.g., the Eurocode Part 1-6-2007, follow the similar procedure implicitly or explicitly. Although using this approach, the presence of imperfections is incorporated, the resulting designs are very conservative. Another approach to

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deal with imperfection sensitivity of thin cylinders is the use of stiffeners along the longitudinal and/or circumferential direction. Though stiffeners, to some degree, reduce the sensitivity to imperfections, they increase the cost of construction. The imperfection sensitivity of thin shells has long been a nuisance to the designers and they just learned to leave with this nuisance. Recent efforts have been trying to re-examine the knockdown factors accounting for realistic dimple imperfections, e.g., Gerasimidis et al (2018), Huhne et al (2008), Wagner et al (2017), Krasovsky et al (2011), Wagner et al (2018), Hilburger et al (2012), Haynie et al (2012) and Kriegesmann et al (2012).

However, recently a novel approach has emerged which gives some hope to get rid of this nuisance called imperfection sensitivity. In this approach, wavy cross-sectional shapes are explored instead of circular cross-sections. The wavy shape cross-section reduces the slenderness (R/t) of the cylindrical shells because the local radius of curvature is reduced and consequently, the imperfection sensitivity of thin cylindrical shells is also reduced. Past studies have been carried out using the wavy shape cross-sections and they present highly promising results for axial compression loading scenarios and elasticity (Xing et al. 2015 and Xing et al. 2017). However, for many important applications, the primary load is bending, e.g., wind turbine towers and gas pipelines. The results of Xing et al. (2015) and Xing et al. (2017) cannot be directly interpolated in these loading scenarios and thus a study of imperfection sensitivity of thin wavy cylinders under bending is needed. Recently, Yadav et al. (2018) have extended the application of wavy cylinders under bending and their findings are very encouraging.

In this paper, imperfection sensitivity of wavy thin cylindrical shells is explored in the inelastic range. First, the effect of imperfections on the bending behavior of wavy and circular thin cylindrical shells is investigated and compared. Next, the effect of the wave parameters, i.e., wave amplitude and the number of waves, on the load reduction factors is studied. The results we found are very encouraging and motivate further studies. Furthermore, an application of wavy cross-sectional thin cylindrical shells in tall wind turbine towers is explored to illustrate the potential benefits of wavy cylinders.

2. Geometry, modeling and material properties of wavy cross-sectional cylindrical shells

In this section, the geometry of the wavy cross-sectional cylindrical shells, material properties, and FEM modeling are described.

2.1 Geometry of wavy cross-sectional cylindrical shells

The cross-section of wavy cylindrical shells is created by modifying the cross-section of circular cylindrical shells. The equation of a circular cross-section, whose center is at the origin, in the polar coordinate is:

$$r(\theta) = R \tag{1}$$

Where r is the distance of the trace point from the center of the cross-section, R is the radius of circle and θ is the angle from the polar-axis as shown in Fig. 1. There are many ways to create wavy cross-sections, we chose a simple equation, given by Xing et al. (2015), to make the wavy cross-section due to its simplicity. In this equation, the wavy cross-section is created by superposing a sinusoidal wave on the equation of the circle. The mathematical expression of the wavy cross-section is:

$$r(\theta) = R + A_r \sin(N\theta) \tag{2}$$

Where r is the distance of the trace point from the center of the cross-section, R is the radius of the circle, θ is the angle from the polar-axis, A_r is the amplitude of sinusoidal wave and N represents the total number of waves. The wavy cross-section is shown in Fig. 1. In this study, the value of R and the ratio of radius to the thickness (R/t) are taken as 2 m and 120 respectively. The length of the cylinder in all the analyses is 20 m, which yields to $\frac{L}{R} = 10$. These values represent a typical section of a tall wind turbine tower (Yadav et al).

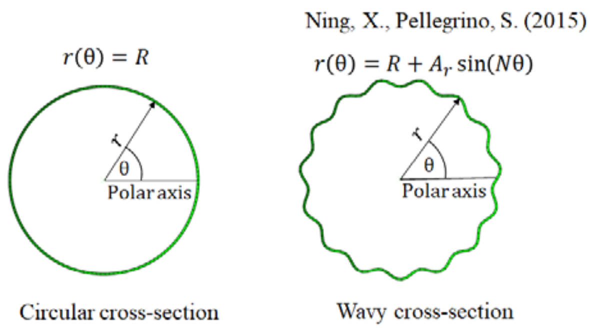

Figure 1: Cross-section of circular and wavy cylindrical shells along with their mathematical expression.

2.2 Stress-strain relationship of the material

Thin steel cylindrical shells, for $\frac{R}{t} = 120$, are expected to fail inelastically and therefore the material hardening model plays a highly important role in their behavior (Kyriakides et al. 2007). We use a version of Ramberg-Osgood stress-strain relationship, which is expressed as (Kyriakides et al. 2007):

$$\varepsilon = \frac{\sigma}{E}\left[1 + \frac{3}{7}\left(\frac{\sigma}{\sigma_y}\right)^{n-1}\right] \tag{3}$$

In the present paper, the value of modulus of elasticity E is 210 Gpa, the Poisson's ratio ν is 0.3, the yield stress σ_y is 355 Mpa and the value of n is 9.

2.3 Geometry of imperfection

To study the effect of geometric imperfections, an axisymmetric geometric imperfection is chosen, whose amplitude is slightly biased toward the center. This imperfection is in the shape of a sinusoidal wave, whose wavelength is 2λ (where λ is the half wavelength of the cylindrical shell under the compressive load whose value is $1.72\sqrt{Rt}$ (Timoshenko et al. 1961)). Mathematically this imperfection can be described as (Kyriakides et al. 2007):

$$w = -R\left(a_{oi} + a_i \cos\left(\frac{\pi x}{\lambda k}\right)\right)\cos\left(\frac{\pi x}{\lambda}\right) \tag{4}$$

Where w represents the deviation from the original position in the radial direction, k is the number of waves along the length, λk is the length of the cylinder whose value is 20 m in this study, $R(a_{oi} + a_i)$ is the amplitude of imperfection and x is the axial coordinate with the origin at the center of the cylinder. Ra_{oi} and Ra_i are the unbiased and biased components of the amplitude respectively. In this study, the value of a_{oi} and a_i are chosen such the ratio $\frac{a_{oi}}{a_i}$ is 5.

2.4 Computational Analyses

For the analyses of cylinders, a displacement-based method of analysis is used with one end of the cylinder is fixed while the rotation is applied at the other free end as shown in Fig. 2. The simulation is performed in ABAQUS by utilizing the Riks method and using four nodes reduced integration shell (*S4R*) elements (ABAQUS). Four integration points are utilized along the thickness of each element. Two rigid body constraints with reference points are imposed at the end cross-sections, which make sure that the end cross-sections do not change their shape, i.e., ovalization is prevented during the analysis. These constraints represent the rigid rings, which are used in cylindrical shells at regular intervals to prevent the ovalization. After performing an extensive mesh convergence analysis, it was found that around 25000 elements provide an element size of 100 mm x 100 mm which is less than $1.72\sqrt{Rt}$. Fig. 2 shows the circular and wavy cylindrical shells along with their boundary conditions and mesh elements.

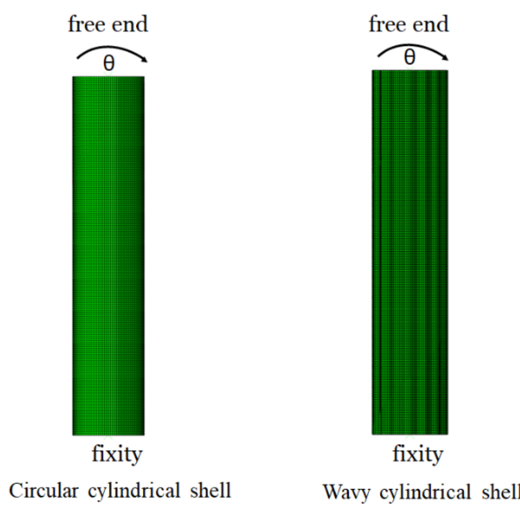

Figure 2: Circular and wavy cylindrical shells along with their boundary conditions.

3. Effect of imperfections

Fig. 3a shows the moment-rotation diagrams of the perfect and the imperfect circular cylindrical shells, and Fig. 3b shows the moment-rotation diagrams of the perfect and the imperfect wavy cylindrical shells. The values of R and R/t for both cylinders (circular and wavy) are 2 m and 120 respectively. For the wavy cylinder, the value of A_r and N are $3R/70$ m and 15 respectively. The amplitude of imperfections (see section 2.3) varies from 0 (perfect) to $2t$ (t is the thickness of cylinders) with an interval of $t/20$ and total 41 models, i.e., one perfect and forty imperfects, are analyzed for both circular and wavy sections. Fig. 3 illustrates very clearly that the difference between the peak moment of the perfect cylinder and the imperfect cylinder is large for the circular cylindrical shells as compared to the wavy cylindrical shells. Thus, the circular cylindrical shell is extremely sensitive to the imperfections while the wavy cylindrical shell is slightly sensitive to the imperfections.

Another significant observation from Fig. 3 is that the moment capacities of wavy cylindrical shells (both perfect and imperfect) are significantly higher than the moment capacities of circular cylindrical shells. For more clarity, the moment capacities of circular and wavy cylindrical shells are drawn in Fig. 4a. Fig. 4a illustrates very clearly the higher moment capacities of wavy cylinders as compared to the circular cylindrical shells. So, the benefits of using wavy cylindrical shells are twofold: first, it reduces the imperfection sensitivity, and second, it also increases the moment capacities.

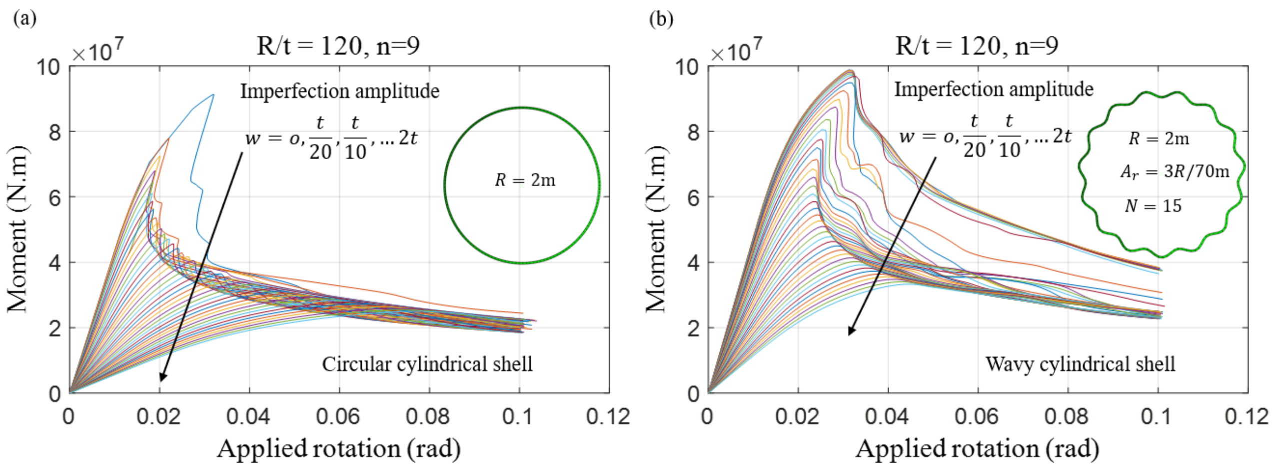

Figure 3: Moment-rotation diagrams of perfect and imperfect (a) circular cylindrical shells and (b) wavy cylindrical shells.

In Fig. 4b, the load reduction factor λ is drawn with respect to the imperfection amplitude (normalized by the thickness t of the cylinder) for the circular and the wavy cylinders. Many important observations could be made from figure 4b.

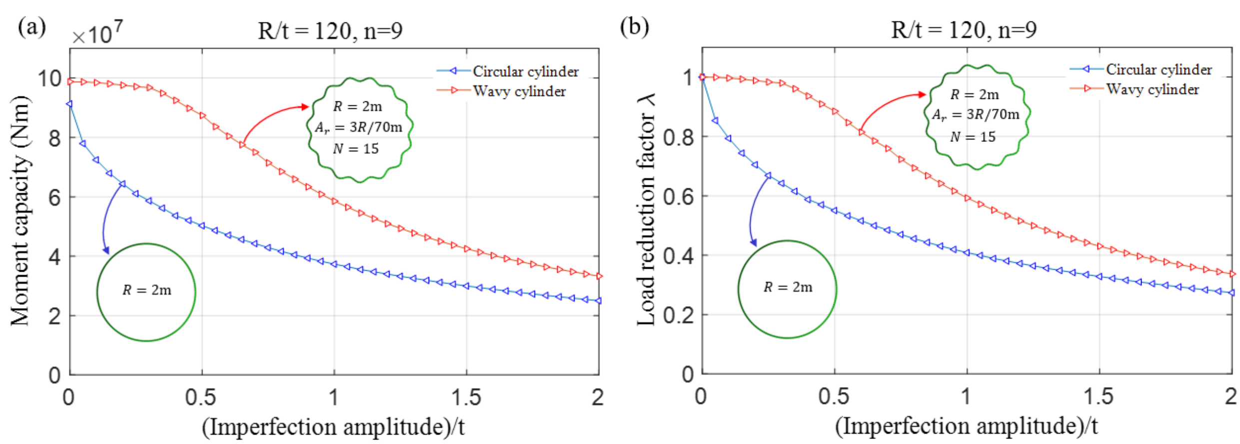

Figure 4: (a) Moment capacities of circular and wavy cylindrical shells. (b) Load reduction factors of circular and wavy cylindrical shells.

First, the load reduction factor λ for the wavy cylinders is always less than the λ of the circular cylinders for all the range of imperfection amplitudes included in this study (form $t/20$ to $2t$). Second, for small imperfection amplitude (imperfection amplitude < $0.3t$) the reduction in load carrying capacity of the wavy cylinders is insignificant while for the circular cylinders the reduction in load carrying capacity is significant. For the imperfection amplitude more than $0.3t$, the reduction in λ becomes visible for the wavy cylinders. The third observation is that the difference between λ for the circular and wavy cylinders is reducing as imperfection amplitude is increasing. The first observation is significant because the amplitude of imperfections is generally less than $0.3t$, and the advantage of wavy cylindrical shells is pertinent in this range.

To study the effect of wave parameters (A_r and N), five different wave amplitudes and five different number of waves are used as shown in Table 1 and total 25 different wavy cylindrical shells are created.

Table 1: The amplitude of waves and the number of waves used in this study to create wavy cylindrical shells.

A_r	N	N	N	N	N
$R/70$	3	5	15	20	25
$2R/70$	3	5	15	20	25
$3R/70$	3	5	15	20	25
$4R/70$	3	5	15	20	25
$5R/70$	3	5	15	20	25

Fig. 5 shows the load reduction factor λ for the five wavy cylindrical shells, which have different number of waves, along with the load reduction factor λ of the circular cylinder. The values of wave amplitude A_r is $\frac{2R}{70}m$.

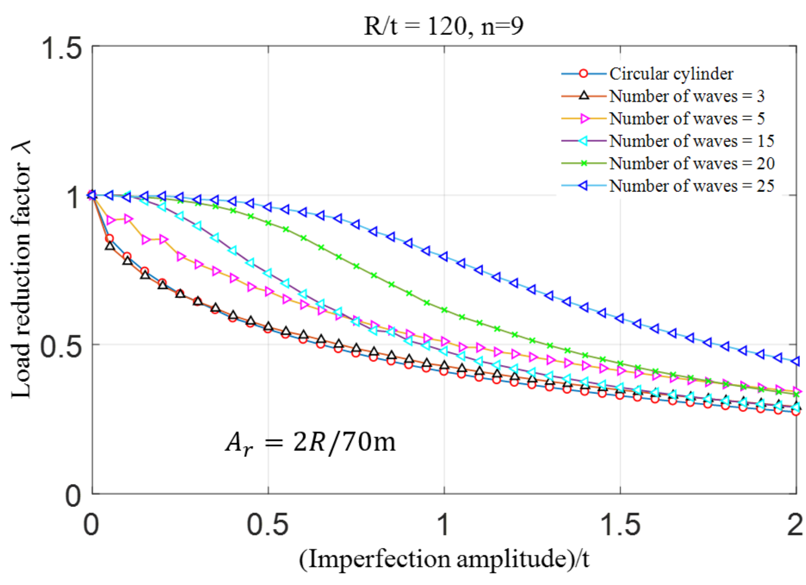

Figure 5: Effect of the numbers of waves along circumferential direction on the load reduction factors. The imperfection sensitivity is reducing with increase of the numbers of wave, the wave amplitude is constant.

From Fig. 5, it can be concluded that the imperfection sensitivity of wavy cylinders reduces as the number of waves increases. For small wave amplitude the load reduction factor λ of the wavy cylinder approaches to the load reduction factor λ of the circular cylinder. This is expected because the circular cylinder is a special case of the wavy cylinder; wavy cylinder becomes a circular cylinder when the wave amplitude reaches to zero.

To study the effect of wave amplitude on the bending behavior of the wavy cylinder, five different amplitudes (i.e., $A_r = \frac{R}{70}, \frac{2R}{70}, \frac{3R}{70}, \frac{4R}{70}, \frac{5R}{70}$) of the wave are chosen as given in table 1. Fig. 7 shows the impact of wave amplitudes on the load reduction factor λ. The load reduction factor λ is significantly affected by the wave amplitude. For the number of waves $N = 15$, the wave amplitude has a big impact on λ if its value is more than $\frac{R}{70}$ while for the wave amplitude less than $\frac{R}{70}$, the load reduction factor λ of the wavy and circular cylinder have slightly differed to each other.

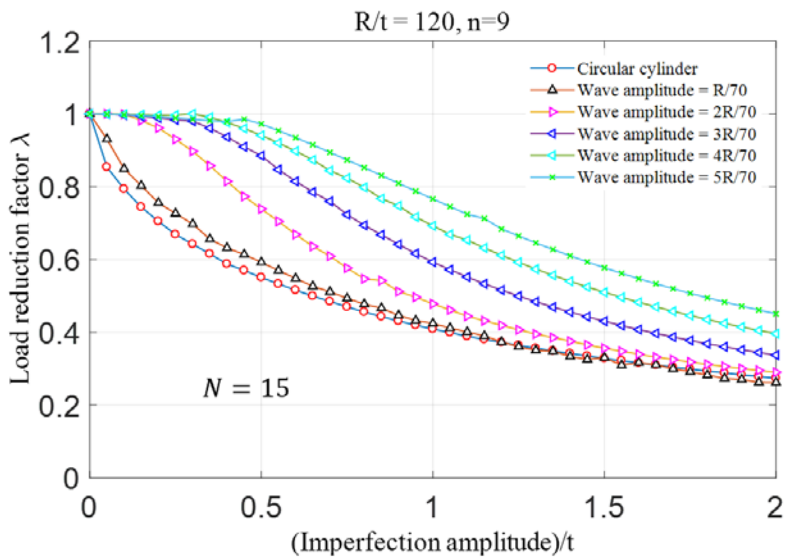

Figure 6: Effect of the wave amplitude on the load reduction factors. The imperfection sensitivity is reducing with increase of the wave amplitude, the numbers of waves is constant.

4. A potential application of thin wavy cylinders for making tall wind turbine towers: A case study

Wind turbines are the critical source of green and clean energy. Their demand has been increased from the last couple of years due to the increased awareness of climate change and the push toward sustainable development. With this increased demand, the need to make them more efficient is also getting an impetus. The efficiency of wind turbines is closely related to the height of their towers as the winds are steady at the higher elevation. This correlation of height and efficiency of wind turbines coupled with their high demand drive engineers to make higher and higher wind turbine towers. Tall wind turbine towers are mostly made by thin cylindrical shells and thus are very sensitive to imperfections. As a result, these towers are designed conservatively to account the presence of imperfections. This increases the cost of wind turbine towers and poses a big obstacle to scaling up the use of wind turbines.

This conservative design of tall wind turbine towers could be avoided if we make the towers insensitive to imperfections. It is possible to make the towers insensitive to imperfections if we use wavy cross-sectional thin cylinders instead of circular cross-sectional thin cylinders for making the towers. In the previous sections, we demonstrated that thin wavy cylinders are insensitive to imperfections as compare to thin circular cylinders under bending. It is interesting to see how a tall wind turbine, made by wavy cylinders, response to imperfections. We study the response of the towers, made by thin circular and thin wavy cylinders, to imperfections. We found that the tower made by wavy cylinders are indifferent to imperfections (for imperfection amplitude less than the thickness of cylinder), while the tower made by circular cylinders are reducing their capacity significantly due to the presence of imperfections. The details about the tower and its response to imperfections are described in the following sections.

4.1 Geometry of the wind turbine tower

The wind turbine tower investigated in this study is a 61 *m* hollow tubular steel tower supporting 1.5 *MW* capacity three-bladed horizontal axis *NORDEX S70/1500* (Sadowski et al. 2017).

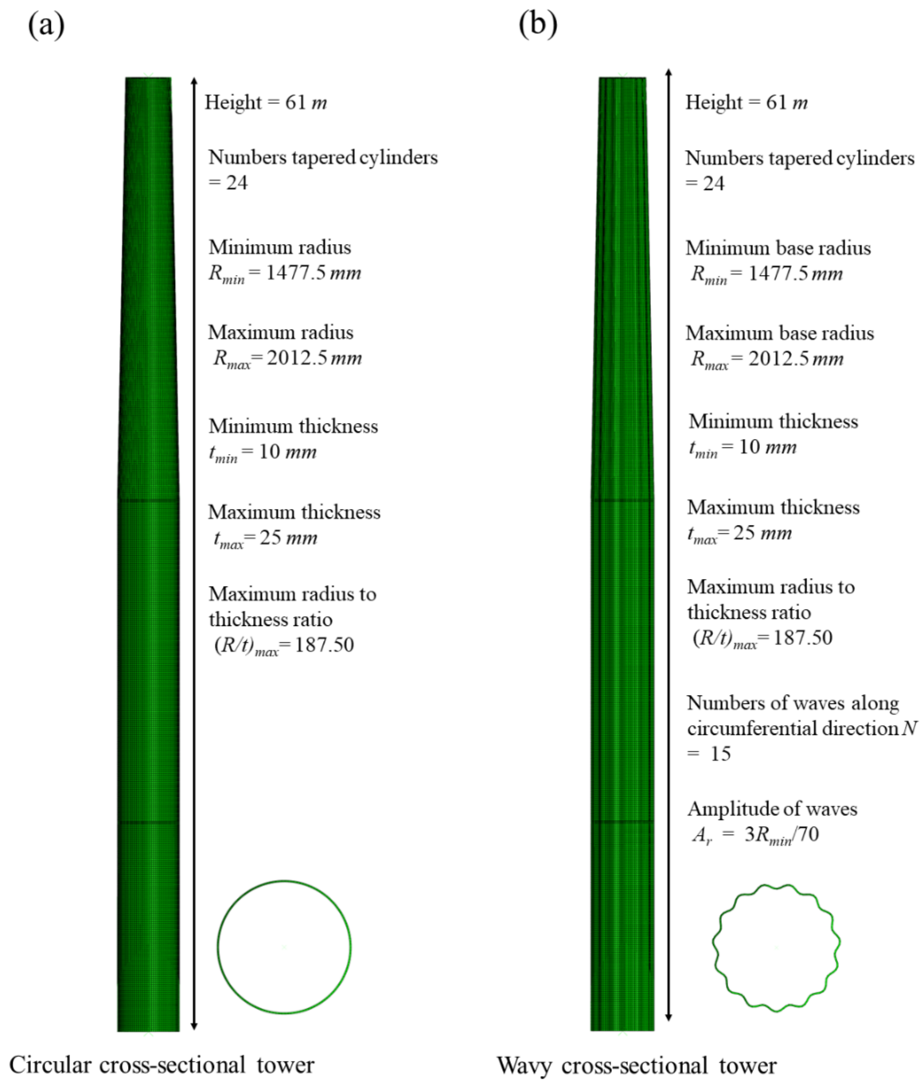

Figure 7: (a) Circular wind turbine tower; it is original prototype. (b) Wavy wind turbine tower, which is created by modifying the original circular tower.

The outer diameter of the tapering tower varied from 4025 *mm* at the base to 2955 *mm* at the top. The thickness of the shell varies from 25 *mm* at the base to minimum 10 *mm* near the top. The diameter to thickness ratio varied from minimum 161 at the base to maximum 375 near the top of the tower. In total 24 tapered thin cylinders are used to create this tower along with three circumferential stiffeners (two in the middle and on at the top). There is a door near the base cylinders, but in this study, we ignore the presence of the door. Fig. 7a shows this circular tapered wind turbine tower.

This wind turbine tower is slightly modified by changing the circular cross-sections to wavy cross-sections of tapered cylinders. The total number of waves along the circumferential direction N is 15 and the amplitude of waves A_r is $\frac{3R_{min}}{70}$ *m.*, Here R_{min} represents the minimum radius of the circular cylinders by which the actual tower is made. These waves are superimposed on the circular tower along the whole length (including the stiffeners to avoid any discontinuity) and a wavy tower is created. Fig. 7b shows the wind turbine tower made by tapered wavy cylinders. For the analysis purpose, we assume that the material used in these towers are same as describe in section 2.2 and follow the strain-stress relationship described by Eq. 3.

4.2 Effect of imperfections on load carrying capacity of towers under bending

The analysis is done in ABAQUS by utilizing the Riks method and using four nodes reduced integration shell (*S4R*) elements. Four integration points are utilized along the thickness of each element. Two rigid body constraints with reference points are imposed at the top and the bottom of the tower, which make sure that the top and bottom do not change their shape. We are interested in the bending capacity of the tower as bending is the primary load on the tower. The rotation is applied at the top and the bottom is fixed. To study the effect of imperfections, an imperfect circular tower and an imperfect wavy tower are created by inducing their respective first eigen mode (under bending) shapes with given imperfection amplitudes as shown in Fig. 8a and 8b.

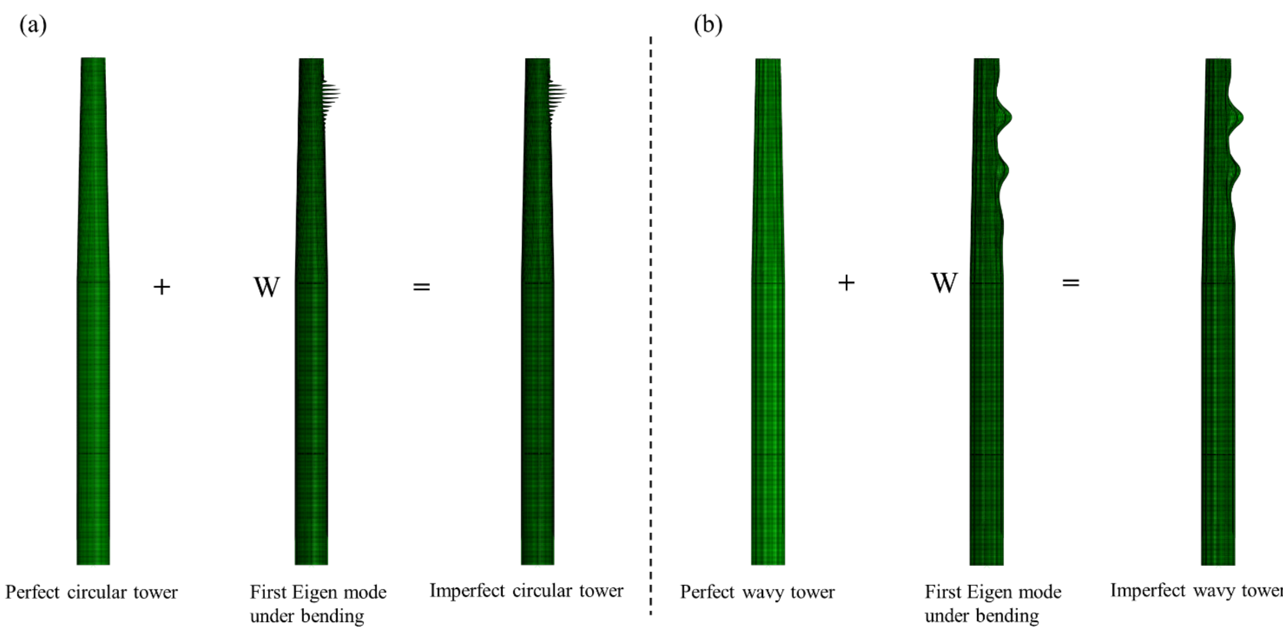

Figure 8: (a) Inducing geometric imperfection in the circular tower. (b) Inducing geometric imperfection in the wavy tower.

We created ten imperfect wavy and circular towers with varying imperfection amplitude and found their capacity. Fig. 9a shows the moment capacities of the imperfect wavy and circular towers with respect to imperfection amplitude. The imperfection amplitude is varied from 0 to t_{min}, where t_{min} is the minimum thickness of cylinders used in making the towers (i.e., 10 *mm*). From Fig. 8a, it is clear that the moment capacity of the wavy tower is higher than the capacity of the circular tower. The difference between the capacities of circular and wavy towers is significant when the imperfection amplitude is more than $0.4t_{min}$. This is due to the indifference of wavy towers to imperfections and high sensitivity of circular tower to imperfections.

Another important feature of Fig. 8a is that for higher imperfection amplitude the capacities are stabilized, i.e., the further increase in imperfection amplitude does not affect the moment capacities. To see the effect of imperfections more clearly, load reduction factor λ is plotted against the normalized imperfection amplitude in Fig. 8b. It should be noted that load reduction factor λ is found by dividing the capacity of the imperfect tower to the capacity of the respective perfect tower. From this figure, it is very clear that the reduction in the bending capacity of the circular tower is significant whereas in case of the wavy tower the reduction is marginal. For imperfection amplitude $0.1t_{min}$, the reduction in the circular tower is around 20% and the reduction in wavy cylinder is around 3.5%. Similarly, for imperfection amplitude $0.4t_{min}$, the reduction in case of the circular tower is around 38% whereas the reduction in case of the wavy tower is around 8%. For imperfection amplitude, more than $0.4t_{min}$, the load reduction factors for both circular and wavy cylinders are almost constant, and a plateau can be seen in Fig. 8b. These observations show very clearly that not just wavy cylinders (as shown in section 3) but also the towers made by tapered wavy cylinders are insensitive to imperfections. In this paper, we just used the respective first eigenmodes of the circular and wavy towers as the imperfection shape. However, we got similar results with other eigenmodes, i.e., the wavy tower is less sensitive to imperfections as compared to the circular tower. These primary results are very encouraging and demand a thorough study, which we are currently doing.

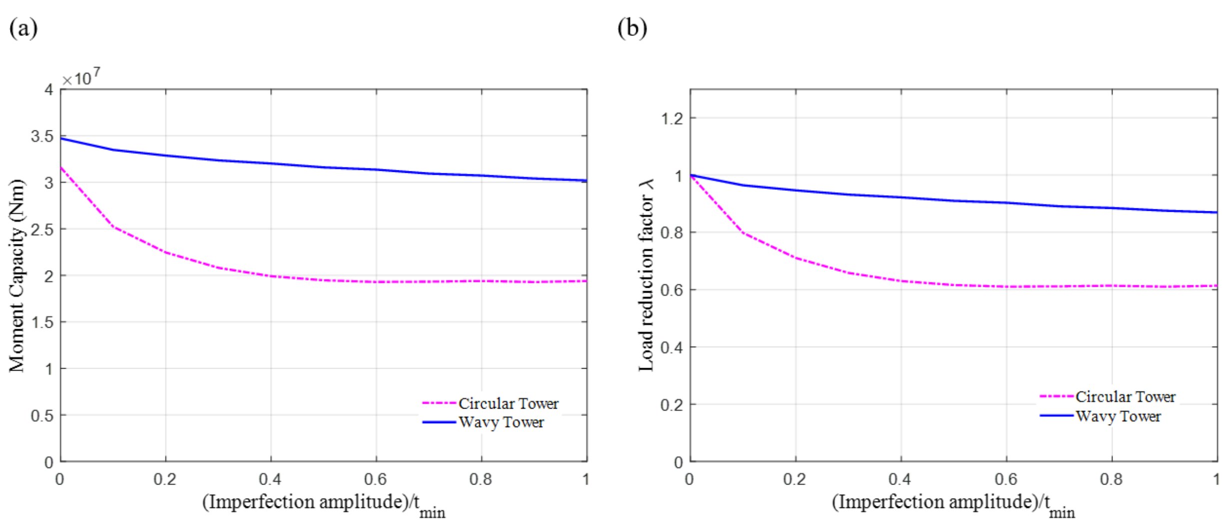

Figure 9: (a) Moment capacities and (b) Load reduction factors of circular and wavy towers against imperfection amplitude.

5. Conclusions

This study aims to understand the bending behavior of thin wavy cross-sectional steel cylindrical shells and to investigate their superiority to thin circular cylindrical shells. We utilized a simple equation to characterize the wavy cylindrical shells; this equation requires three parameters, e.g., the radius R, the wave amplitude A_r, and the number of waves N to fully define the cross-section of wavy cylindrical shells. To understand the bending behavior of thin wavy cylindrical shells, Finite Element Method is performed using the commercial software ABAQUS. The rotation is applied at the one end of the cylinder while the other end is fixed. For the analyses purpose, a version of the Ramberg-Osgood model is used as the cylinder is expected to fail inelastically for the given R/t ratio.

This study reveals that wavy cylindrical shells under bending are insensitive to the imperfections and the presence of imperfections does not reduce the load carrying capacity significantly. It is an important finding for cylinders under bending. Previous studies (Xing et al. 2015 and Xing et al. 2017) have revealed similar conclusions but for the thin cylindrical shells under the axial compression. Comparison between the load reduction factor λ of the wavy cylinders and the circular cylinders is also performed and we found that for the small amplitude of imperfection ($w < 0.3t$) the reduction in bending capacities is high for the circular cylinders compared to the insignificant reduction for the wavy cylinders. Apart from this, we also found that the bending capacities of wavy cylinders are higher than the bending capacities of the circular cylinders. To investigate the effect of wave amplitude (A_r) and the number of waves (N), a parametric analysis is performed. It is found that the effectiveness of the wavy cylinder increases as the number of waves and wave amplitude increase. This means that for the high wave amplitude and the high number of waves, wavy cylinders are less sensitive to imperfections, and their bending capacities are high.

To explore potential applications of wavy thin cylindrical shells, an actual tall wind turbine tower, which is made by thin tapered circular cylinders, is modified slightly and a new tower is created, which is made of wavy tapered thin cylinders. We make these towers, i.e., circular and wavy, imperfect by introducing their respective first eigen modes under bending with given amplitudes. We found that the wavy tower is insensitive to imperfections whereas circular tower (the actual tower) is highly sensitive to the imperfections and loss its significant capacity due to the presence of imperfections. This finding, although preliminary in nature, demonstrates that the capacity of wind turbine tower can be increased, and imperfection sensitivity can be reduced if wavy cylinders are used to make them.

These results are very promising because imperfection sensitivity of thin cylindrical shells has been a big obstacle for their economic applications for a long time. This study shows that if wavy cross-sectional cylindrical shells are used, the inherent drawback (high imperfection sensitivity) of thin circular cylindrical shells can be circumvented and material can be used optimally. This study is limited in many respects: the manufacturing feasibility of wavy cross-sectional thin cylindrical shells has not been studied, the specific R/t ratio (120) is used, and the cost comparison of wavy cylinders with circular cylinders has not been done. Nevertheless, this study divulges an important aspect of thin wavy cylindrical shells and in the future other territories of thin wavy cylindrical shells can be explored.

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