Proceedings of the
Annual Stability Conference
Structural Stability Research Council
St. Louis, Missouri, April 2-5, 2019

Time-Dependent Buckling of Steel Plates Exposed to Fire

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Abstract
This paper reports on a preliminary analytical study examining the time-dependent behavior of steel plates subjected to fire temperatures. Simply-supported rectangular plates under uniaxial compression are considered. Isochronous stress-strain curves are utilized to approximate the time-dependent behavior of steel at elevated temperatures. The creep model by Fields and Fields for ASTM A36 steel is used to construct isochronous stress-strain curves. Using the concept of time-dependent tangent modulus, closed-form formulas are developed to model the creep buckling of steel plates at elevated temperatures. These nonlinear equations are evaluated to visualize the time- and temperature-dependent behavior of steel plates in the form of *Creep Buckling Curves* and *Isochronous Buckling Curves*. The creep buckling expressions are further utilized to study the effect of width-to-thickness ratio (slenderness ratio) on the creep buckling phenomenon of steel plates at elevated temperatures. Results from creep buckling analyses presented in this paper indicate that the buckling strength of plates made of ASTM A36 steel can be highly time-dependent for temperatures at or above 500 °C. The results further show that thermal creep of steel has a more significant impact on the behavior of shorter steel plates, whose buckling strength is governed by stability in the inelastic range.

1. Introduction

Understanding the buckling behavior of plates at elevated temperatures is essential in predicting the local buckling behavior of steel plate assemblies in fire. One of the critical factors affecting the behavior of steel plates in fire is the thermal creep of structural steel. Thermal creep of steel results in additional deflection of plates, and as a result, additional moment. Consequently, the behavior of steel plates at elevated temperatures is affected by a phenomenon called creep buckling, where the buckling capacity of a steel plate depends on the duration of applied stresses in addition to the slenderness and temperature. The effect of thermal creep of steel on the buckling behavior of steel plates exposed to fire is therefore of particular importance, and it is not well understood and clearly treated in building codes and standards.

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The analysis of the buckling behavior of steel plates under in-plane compression at room temperature is well documented in the literature on the stability of steel plates (Timoshenko 1936; Bleich 1952; Gerard and Becker 1957; Ziemian 2010). However, literature on the buckling capacity of steel plates exposed to fire temperatures is quite meager (e.g. Quiel and Garlock 2010). Specifically, there is no study on the effect of thermal creep of steel on the buckling behavior of steel plates when subjected to fire (limited literature exists on the creep buckling of stainless steel and aluminum plates for aircraft industry application, e.g. Gerard 1962). The time-dependent or creep behavior of steel plates exposed to fire is therefore not well understood and clearly treated in building codes and standards including Eurocode 3 (2003) and the AISC Specification (2016).

To develop a better understanding of the effect of creep of steel on the buckling behavior of steel plates at elevated temperatures, the authors performed an analytical study, highlights of which are presented in this paper. The concepts of isochronous stress-strain representation of the thermal creep of steel, and the time-dependent tangent modulus to model the creep buckling of steel plates at elevated temperatures are first introduced. Results from both short-time and creep buckling analyses on PL3.2×16×0.2 plate made of ASTM A36 steel under in-plane compression at elevated temperatures are then presented and discussed. Conceptual representations of the phenomenon of creep buckling of steel plates in the form of *Creep Buckling Curves* and *Isochronous Buckling Curves* are further proposed to better understand and quantify the combined effects of compressive stresses and plate slenderness ratios on the time- and temperature-dependent behavior of steel plates.

2. Creep of Steel at Elevated Temperatures

It is generally accepted that for ductile materials like steel, plastic strain is a function of shear stress and time at any specific temperature. Therefore, for design purposes, it is usually assumed that the total plastic strain at a constant temperature can be broken into a *time-independent component* or *slip* and a *time-dependent component* or *creep*. For typical loading rates seen in buildings, the inelastic response of steel at room temperature shows a very mild dependence on loading rate and virtually no dependence on time. Therefore, time effects are normally neglected in the analysis and design of steel structures at ambient temperature. However, as temperature increases, steel exhibits increasingly significant creep effects.

Creep tests, either in tension or compression, are usually conducted by subjecting a material to constant load, hence constant engineering stress at a specific temperature, and then measuring engineering strain as a function of time. A typical creep curve is often divided into the three stages of primary, secondary and tertiary creep. In the primary stage, the curve is nonlinear and typically exhibits a decreasing creep strain rate with increase in time. In the secondary stage, the creep strain rate is almost constant, and this stage is often referred to as steady-state creep. In the tertiary stage, the creep strain rate increases with time in an unstable manner. For steel, the shape of the curve, the magnitude of the creep strain and the time scale are greatly affected by both the temperature and the stress level.

Experimental and empirical models have been developed to predict creep strain of steel at elevated temperatures (Norton 1929; Bailey 1929; Zener and Hollomon 1944; Dorn 1955; Harmathy 1967; Fields and Fields 1989). One of the simplest and most widely used creep models is the Norton-Bailey model, also known as the creep power law (Norton 1929; Bailey 1929). One of the widely used creep models in structural-fire engineering applications proposed by Fields and Fields (1989) incorporates a power law and represents creep strain, ε_c, in the form of a Norton-Bailey equation as follows:

$$\varepsilon_c = at^b\sigma^c \tag{1}$$

In Eq. 1, t is time and σ is stress. The parameters a, b and c are temperature-dependent material properties. Fields and Fields (1989) derived equations for these temperature-dependent material properties for ASTM A36 steel. The model developed by Fields and Fields (1989) is capable of predicting creep in the temperature range of 350 °C to 600 °C and for creep strains up to 6-percent. For initial studies of creep buckling of steel plates at elevated temperatures, one of the creep models used by the authors was the Fields and Fields (1989) model. The application of this creep model together with observations will be discussed in more detail in the following sections of this paper.

3. Creep Buckling of Steel Plates at Elevated Temperatures

3.1 Background on Creep Buckling

The term *creep buckling*, as used herein, refers to the phenomenon in which the critical buckling stress for a plate depends not only on width-to-thickness ratio (referred to as the slenderness ratio in this paper) and temperature of the plate, but also on the duration of applied stress. Since creep effects are not significant at room temperature, the buckling stress for a steel plate of given effective slenderness b/h (where b is the width, and h is the thickness of the plate) at room temperature is independent of the duration of applied stress. As temperature increases, the initial buckling stress (at time zero) decreases, due to the decrease in material strength, modulus and proportional limit. Consequently, the buckling capacity at initial application of stress depends only on temperature. However, as temperature increases and material creep becomes significant, the buckling stress depends not only on temperature, but also on the duration of stress application.

3.2 Creep Buckling Analysis of Steel Plates

To better evaluate the potential importance of thermal creep in predicting buckling strength of steel plates at elevated temperatures, the authors have conducted preliminary creep buckling analyses. These analytical analyses attempt to predict the elevated-temperature creep buckling strength of simply-supported steel plates under uniaxial compression. For these analyses, a PL3.2×16×0.2 plate made of ASTM A36 steel is considered. Moreover, the effective slenderness ratio is kept constant by considering only one single plate thickness of 0.2 inches.

For the analytical creep buckling studies, the concept of time-dependent tangent modulus proposed by Shanley (1952) is utilized, along with the creep material models developed by Fields and Fields (1989) for ASTM A36 steel. This analytical method basically uses the Elastic buckling equation and replaces Young's Modulus, E, with the tangent modulus, E_T, which is a function of time, stress and temperature. In order to calculate the time-dependent tangent modulus, the isochronous stress-strain curves need to be constructed. Simply put, isochronous stress-strain curves are constant-time stress-strain curves derived from creep curves. The slope of the tangent to the isochronous stress-strain curve at any stress and time value is the time-dependent tangent modulus. The procedure of constructing isochronous stress-strain curves using the material creep model by Fields and Fields (1989) and evaluating time-dependent tangent moduli correspondingly are explained in the following. At a specific time, Eq. 1 can be rewritten as follows,

$$\varepsilon_c = a_o \, \sigma^{\,c} \tag{2}$$

where a_o is equal to at^b and is constant. In fact, since a_o is dependent on a, b and t, it is both temperature and time dependent. It can also be inferred from Eq. 2 that each constant-time, stress-creep strain curve is conceptually equivalent to a time-independent stress-plastic strain curve, here with the power law representation (Morovat et al. 2013, and Morovat 2014). As a result, the total strain, which is the sum of elastic, plastic (time-independent inelastic) and creep (time-dependent inelastic) strains can be written as,

$$\varepsilon = \frac{\sigma}{E} + f\sigma^{\,g} + a_o \, \sigma^{\,c} \tag{3}$$

The second term on the right-hand side of Eq. 3 represents the plastic or time-independent inelastic strain. For the plastic strain, the terms f and g are again temperature-dependent material parameters. Fields and Fields (1989) also derived equations for these temperature-dependent material parameters for ASTM A36 steel. Eq. 3 therefore represents the isochronous stress-strain curves based on the creep model by Fields and Fields (1989). Representative isochronous stress-strain curves based on Eq. 3 at 400 °C are shown in Fig. 1.

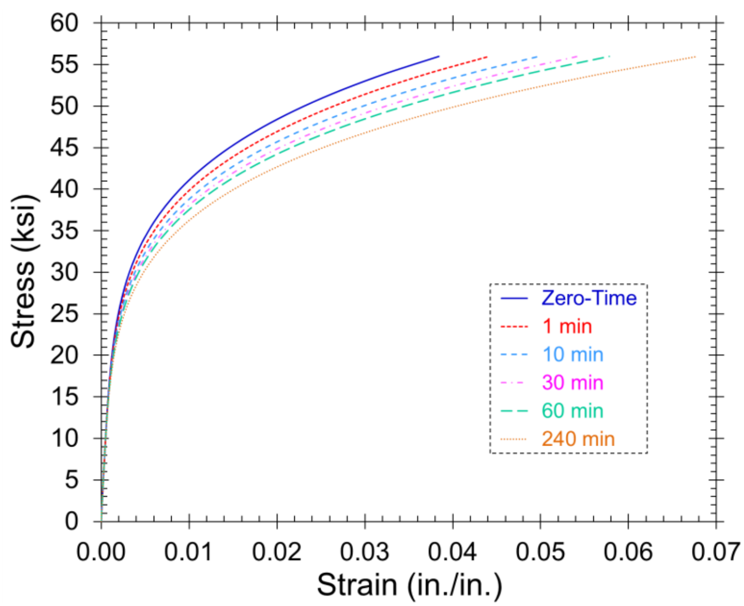

Figure 1: Representative Isochronous Stress-Strain Curves at 400 °C

Eq. 3 can be further used to derive an expression for time-dependent tangent modulus. In other words, using the differential form of Eq. 3. and considering the tangent to be the slope of the stress-strain curve, $d\sigma/d\varepsilon$, a mathematical expression relating tangent modulus to stress can be derived as follows,

$$E_T = \frac{E}{1 + \left[fg\sigma^{(g-1)} + a_o c\sigma^{(c-1)} \right] E} \tag{4}$$

in which, E is the temperature-dependent Young's modulus and E_T is the tangent modulus, here a function of both time and temperature. Representative isochronous tangent modulus-stress curves based on Eq. 4 at 400 °C are shown in Fig. 2.

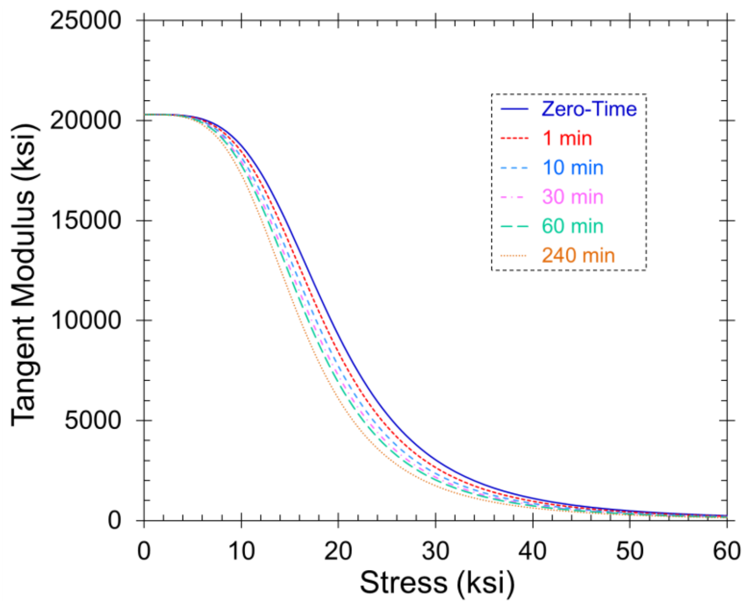

Figure 2: Representative Isochronous Tangent Modulus-Stress Curves at 400 °C

Isochronous tangent modulus-stress curves constructed using Eq. 4 can be used to determine creep buckling stresses graphically. From the classical tangent modulus theory for inelastic plate behavior (e.g. Bleich 1952), the relationship between stress and tangent modulus at a specific temperature can be written as,

$$\sigma = \left[\frac{k\pi^2}{12\,(1-\upsilon^2)\left(\frac{b}{h}\right)^2} \right] E_T \quad \text{or} \quad E_T = \left[\frac{12\,(1-\upsilon^2)\left(\frac{b}{h}\right)^2}{k\pi^2} \right] \sigma \tag{5}$$

Note that, in Eq. 5, k is plate buckling coefficient (equals to 4 for simply supported plates), υ is Poisson's ratio of steel (considered as a constant value of 0.3 regardless of the temperature), b is the plate width, and h is the plate thickness. The term, b/h, is commonly referred to as the width-to-thickness ratio (b/h is referred to as the slenderness ratio in this paper). From Eq. 5, it can be deduced that constant slenderness ratios represent straight lines through the origin on the tangent modulus-stress plots. Intersections of such lines with each tangent modulus-stress isochrone have horizontal components on the stress axis. These stress components are therefore time-dependent buckling stresses for the plate in consideration. In addition, the time isochrone corresponding to each specific creep stress and consequently the creep buckling stress is referred to as the failure time or time-to-buckle.

The process of graphical evaluation of creep buckling stresses is further illustrated in Fig. 3, where two straight lines associated with two different slenderness ratios for a PL3.2×16 plate are shown along with isochronous tangent modulus-stress curves determined using Eq. 4 at 500 °C. As an example, for the failure time of 240 minutes in Fig. 3, the creep buckling stress of about 19.2 ksi is predicted for the PL3.2×16 plate with the thickness of 0.20 inches (*b/h* ratio of 16).

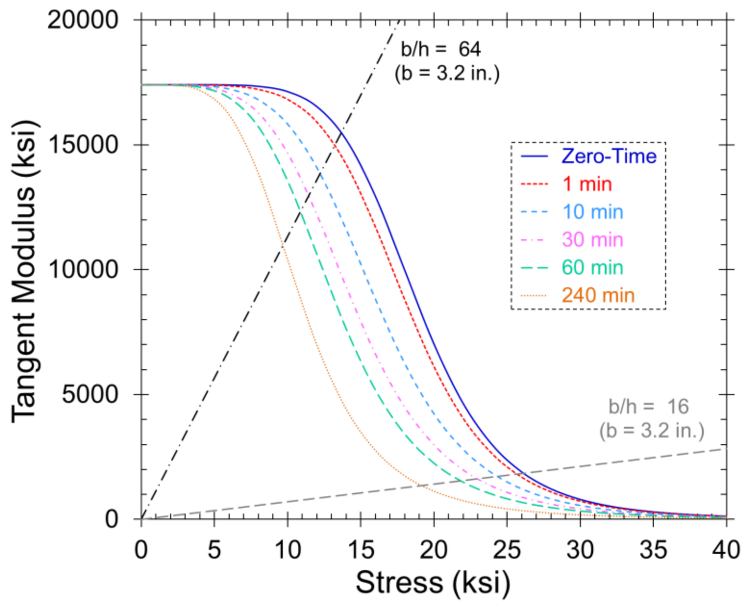

Figure 3: Graphical Representation of the Concept of Creep Buckling of Steel Plates at 500 °C

In addition to the graphical solution described above, Eq. 4 can be used to obtain creep-buckling curves numerically. Since $E_T / E = \sigma_{cr} / \sigma_E$, Eq. 4 yields an equation for creep buckling, which is shown as Eq. 6.

$$\sigma_{cr} = \frac{\sigma_E}{1+\left[fg\sigma^{(g-1)}+a_o c\sigma^{(c-1)}\right]E} \tag{6}$$

σ_E is the Elastic buckling stress at elevated temperatures in Eq. 6. At buckling, $\sigma = \sigma_{cr}$, therefore Eq. 6 can be rewritten as follows,

$$\sigma_{cr} + fgE\sigma_{cr}^{g} + a_o cE\sigma_{cr}^{c} = \sigma_E = k\,\frac{\pi^2 E}{12\,(1-\upsilon^2)\left(\frac{b}{h}\right)^2} \tag{7}$$

Eq. 7 can be solved iteratively to get the σ_{cr} as a function of time (or time-to-buckle) at a constant temperature. Sample solutions of Eq. 7 applied to a PL3.2×16 plate with the thickness of 0.2-inches (*b*/*h* ratio of 16) are shown in Fig. 4. Fig. 4 plots the time-dependent behavior of this plate as graphs of buckling stress versus time for representative temperatures of 400 °C, 500 °C, and 600 °C. As can be seen, as temperature increases, the buckling strength of the steel plate in consideration becomes more time-dependent. More specifically, the buckling capacity of the steel plate drops more rapidly with time at temperatures equal or greater than 500 °C. This can be observed more clearly in Fig. 5, where buckling stresses are shown as normalized with respect to the buckling stress at time zero for any specific temperature. As will be discussed in the next section, the graphs of buckling stress versus time shown in Fig. 4 are referred to as *Creep Buckling Curves* in this paper.

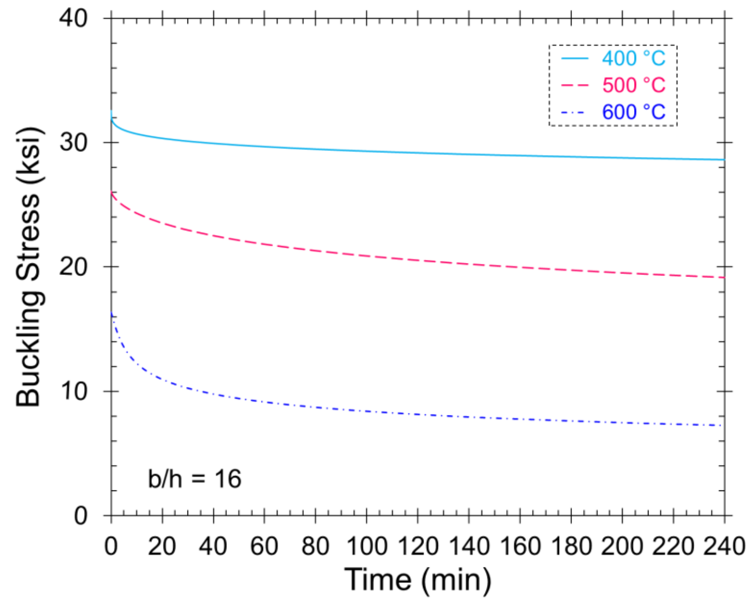

Figure 4: Analytical Creep Buckling Curves using the Concept of Time-Dependent Tangent Modulus

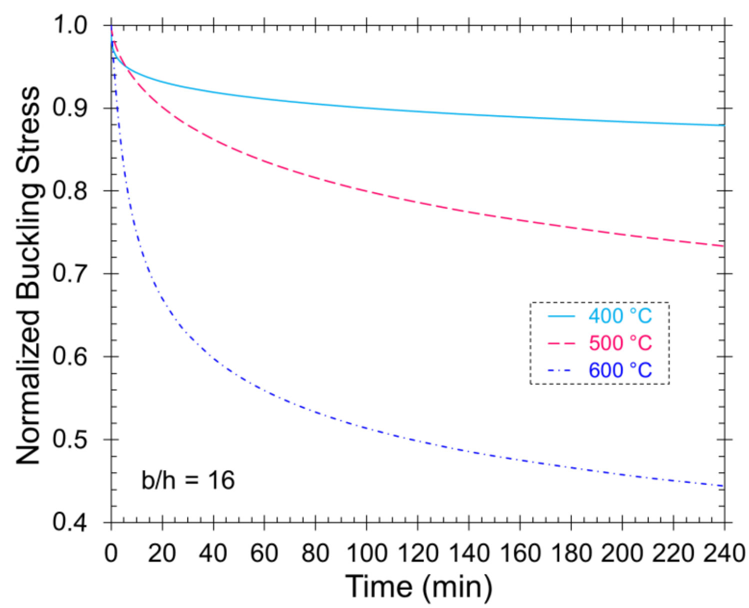

Figure 5: Analytical Normalized Creep Buckling Curves using the Concept of Time-Dependent Tangent Modulus

4. Effect of Slenderness Ratio

In the analysis presented in Section 3, the effect of thermal creep of structural steel on predictions of the buckling strength of steel plates subjected to fire temperatures was visualized through curves of buckling stress versus time (time-to-buckle) for a specific plate. To develop a more comprehensive understanding of the time- and temperature-dependent buckling behavior of steel plates, plates with different slenderness ratios need to be considered. The effect of slenderness ratio on the creep buckling strength of steel plates at elevated temperatures can be represented using the concepts of *Creep Buckling Curves* and *Isochronous Buckling Curves*.

4.1 Creep Buckling Curves

Using Eq. 7, a series of buckling stress versus time graphs representing the creep buckling behavior of steel plates with different slenderness ratios can be generated. Examples of such curves, referred herein as *Creep Buckling Curves*, are shown in Fig. 6 for the representative temperature of 500 °C. Each point on these curves represents a creep buckling test performed using Eq. 7, and as a result, values on the time axis are indicative of *failure time* or *time-to-buckle*. Further, results from short-time (time-independent) buckling analyses are indicated as buckling stresses at time zero on the creep buckling curves in Fig. 6. As expected and seen in Fig. 6, the plate slenderness ratio significantly influences the time-dependent strength of plates at the elevated temperature of 500 °C.

To better understand the effect of slenderness ratio on the time-dependent response of steel plates at elevated temperatures, the creep buckling curves in Fig. 6 are normalized with respect to the corresponding short-time buckling stresses and replotted in Fig. 7. As seen more clearly in Fig. 7, the buckling capacity of steel plates with larger slenderness ratios is more affected by the thermal creep of steel at the representative temperature of 500 °C. This trend continues for slenderness ratios in the order of 64. For slenderness ratios equal or greater than 64, the behavior of plates is governed by the Young's Modulus of steel and therefore is less influenced by the thermal creep of steel (note that the modulus of elasticity of steel is assumed time-independent in this study). For instance, as seen in Fig. 7, buckling strength of the steel plate with slenderness ratio of 128 is not affected by the thermal creep of steel at 500 °C.

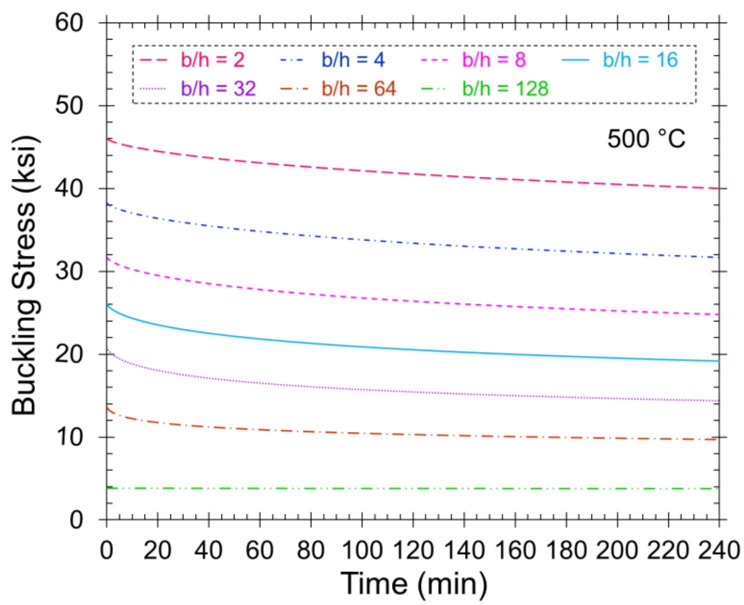

Figure 6: Representative Creep Buckling Curves for Steel Plates at 500 °C:
Effect of Slenderness Ratio

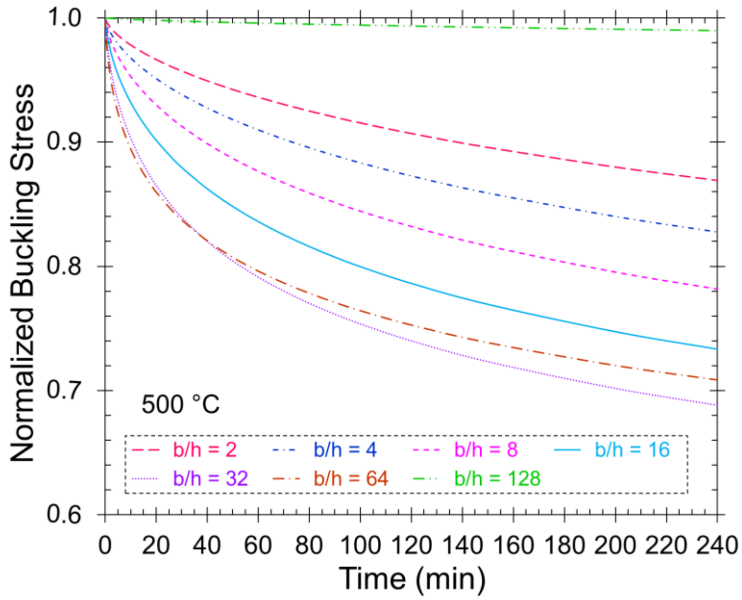

Figure 7: Representative Normalized Creep Buckling Curves for Steel Plates at 500 °C: Effect of Slenderness Ratio

4.2 Isochronous Buckling Curves

In all analyses presented so far, the effect of thermal creep of structural steel on predictions of the buckling strength of steel plates subjected to fire temperatures is visualized through a plot of buckling stress versus time (time-to-buckle). Time effects on the buckling capacity of steel plates can alternatively be illustrated in the form of *Isochronous Buckling Curves*, a sample of which depicted in Fig. 8. Fig. 8 basically presents a series of buckling stress-plate thickness isochrones for simply supported plates under the axial compression at 500 °C. Each isochrone represents a buckling strength curve corresponding to a specific buckling time. Consequently, the isochronous buckling curves in Fig. 8 can be used to predict the time-dependent ultimate strength of PL3.2×16 plates with different thicknesses at 500 °C. Note that this presentation of creep buckling behavior of steel plates closely resembles the isochronous stress-strain behavior of the plate material at the corresponding temperature, and therefore, can be utilized to study the direct impact of thermal creep of steel on the time-dependent buckling behavior of steel plates subjected to fire.

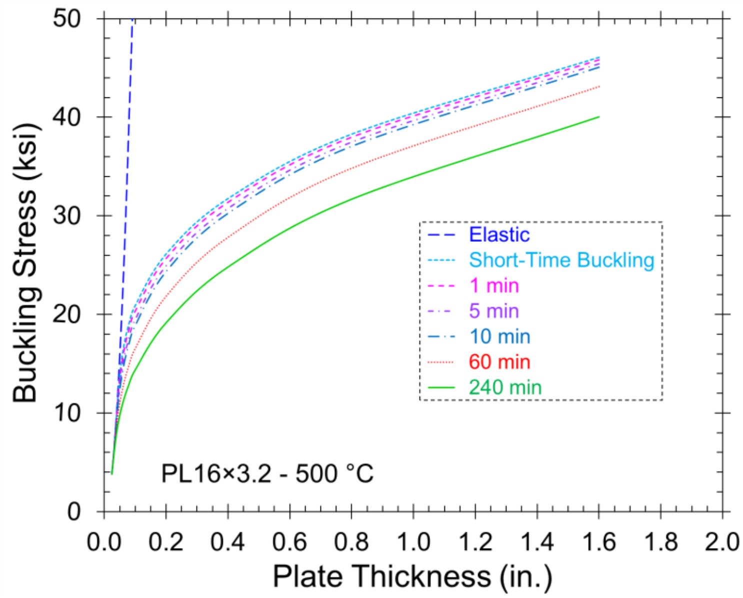

Figure 8: Isochronous Buckling Curves for Simply Supported Plates at 500 °C: Stress vs. Plate Thickness

The effect of thermal creep of steel on the buckling behavior of steel plates subjected to fire can also be presented in the conventional method of plotting buckling stress versus slenderness ratio, commonly referred to as the buckling curve. This can be achieved by constructing isochronous buckling stress-slenderness ratio curves, representative of which are shown in Fig. 9. As seen in Fig. 9, the shape of buckling curves changes with time even for short exposure times. It can be further observed that time effects become more significant for plates with *b/h* in the range of 10 to 50, a range representative of plate elements of the majority of wide-flange sections used in steel structures.

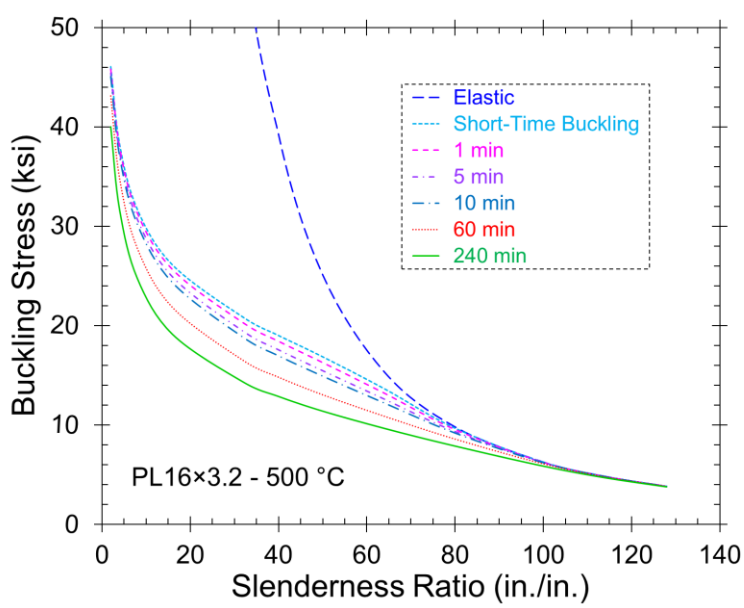

Figure 9: Isochronous Buckling Curves for Simply Supported Plates at 500 °C: Stress vs. Plate Slenderness Ratio

5. Conclusion

Highlights of an extensive analytical study conducted by the authors to investigate the effect of creep of steel on the buckling behavior of steel plates subjected to fire were presented in this paper. Both time-independent and creep buckling analyses of a PL3.2×16×0.2 plate made of ASTM A36 steel under the simply supported boundary condition were presented to characterize short-time and creep buckling stresses at elevated temperatures.

The results from creep buckling simulations indicate that the magnitude of the applied stress and the plate slenderness ratio have a significant impact on the predictions of the creep buckling strength of steel plates subjected to fire. Representations of the time- and temperature-dependent behavior of steel plates in the forms of *Creep Buckling Curves* and *Isochronous Buckling Curves* help to better understand and quantify the effects of axial stress and slenderness ratio on the overall stability of steel plates subjected to fire temperatures.

Acknowledgments

The research reported herein was made possible through previous funding from the National Science Foundation for research projects on *Elevated Temperature Performance of Beam End Framing Connections*, on *Creep Buckling of Steel Columns Subjected to Fire* and on *Elevated Temperature Performance of Shear Connectors for Composite Beams*, (NSF Awards 0700682, 0927819 and 1031099, respectively). The support of the National Science Foundation and of the former NSF Program Directors M.P. Singh and Douglas Foutch is gratefully acknowledged. Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the authors and do not reflect the views of the National Science Foundation.

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