conference preview

STABILITY ANALYSIS AND DESIGN

A look at the AISC 360-10 Chapter C stability provisions and the role of the K-factor in stability design.

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THE 2010 AISC *SPECIFICATION* provides three methods of design for stability: the direct analysis method, the effective length method and the first-order analysis method. While they are all based on the same fundamental principles, it is important to understand their individual characteristics.

So what are the factors that affect stability? Section C1 of the *Specification* requires designers to consider each of the following when investigating stability, regardless of which stability design method is used:

- First-order effects
- Second-order effects
- Geometric imperfections
- Residual stresses
- Uncertainty in stiffness and strength

The above items are considered in each of the three stability design procedures. The means by which they are considered varies, but in all cases they are accounted for either in the second-order analysis or in the computation of available member strength.

First-order effects. First-order effects are the moments, shears, axial loads and deformations resulting from externally applied forces such as gravity, wind and seismic forces and from initial imperfections in the structure.

Second-order effects. Second-order effects are the additional moments, shears, axial forces and deformations that occur as a result of first-order deformations.

There are two types of second-order effects: P-Δ and P-δ effects. P-Δ effects are those occurring due to relative deformations between the ends of columns, and P-δ effects are those occurring due to deformations within the column length. In most cases P-Δ effects are the result of lateral loads acting on the structure and P-δ effects are the result of column moments due to gravity loads (see Figure 1).

Steel-framed structures have historically possessed sufficient inherent stiffness such that second-order effects were of little consequence. Today's use of higher-strength steel, open floor plans, lighter-weight materials and refined design practices have allowed for the design of lighter, more highly stressed, slender and flexible structures for which second-order effects may be of greater consequence. Façade systems and other non-structural components may likewise be sensitive to additional lateral drift due to second-order effects. Accordingly, the *Specification* requires mandatory consideration of second-order effects for all structures.

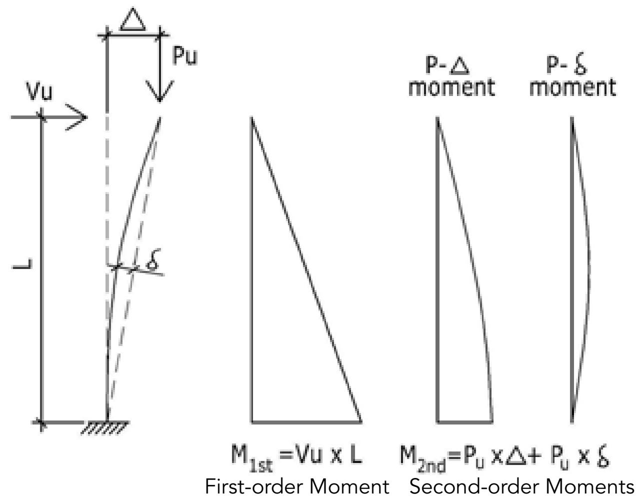

Figure 1. Illustration of first-order and second-order moments in a cantilevered column.

Geometric imperfections. Geometric imperfections can exist in structures due to sweep or camber in steel shapes along their length, imperfections in cross-section geometry and fabrication and erection tolerances, resulting in columns being out-of-plumb. These imperfections can adversely affect stability.

Permissible fabrication tolerances are specified in the 2010 AISC *Code of Standard Practice for Steel Buildings and Bridges* Section 6.4; erection tolerances are specified in Section 7.13. Columns are permitted to be out-of-plumb by as much as *L*/500 (subject to an upper limit). The effects of column out-of-plumbness imperfections are quantified by applying fictitious notional lateral loads to replicate the destabilizing effects occurring from out-of-plumb columns (see Figure 2).

The direct analysis method permits designers the option of directly modeling initial imperfections as an alternative to using notional loads.

Residual stresses. Uneven cooling, which occurs during the manufacture of hot-rolled steel shapes, creates internal residual

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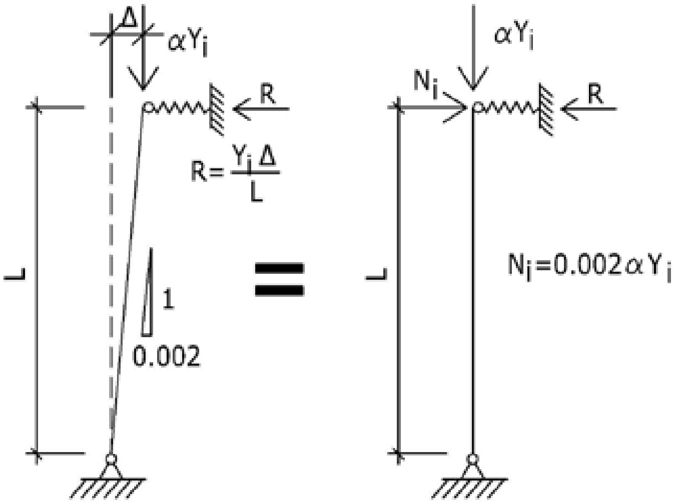

Figure 2. Notional loads are used to model destabilizing effects resulting from of out-of-plumb columns.

tension and compression stresses within the shapes. The distribution of residual stresses varies depending on geometry, rolling temperature, rate of cooling and yield strength. The tips of W-shape column flanges are typically in compression (see Figure 3). Residual stresses affect *both* global stability and column strength.

Figure 4 shows a typical stress-strain curve for structural steel. When stresses reach F_y, E=0 and the steel loses all stiffness.

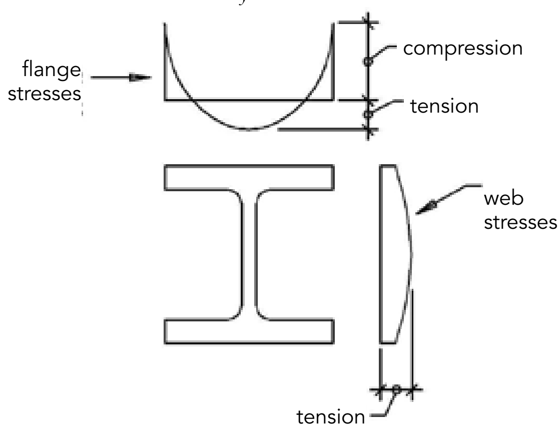

Figure 3. Approximate distribution of residual stresses in a heavy W14 column section.

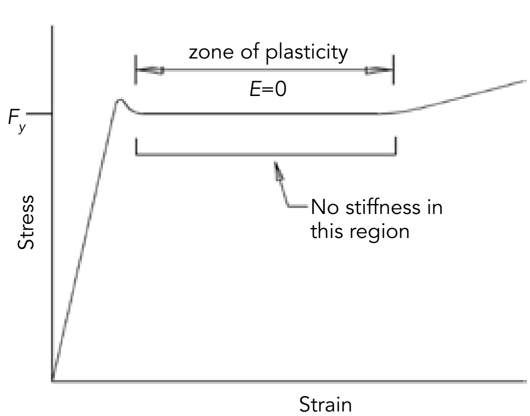

Figure 4. Stress-strain curve for typical structural steel.

As compression stresses increase due to applied loads, yielding will first occur where residual compression stresses are highest—generally near the tips of column flanges. Accordingly, premature yielding where compressive residual stresses are high results in loss of column stiffness (see Figure 5). This loss of stiffness reduces both column strength and flexural stiffness, thus adversely affecting stability by reducing the stiffness of moment frames.

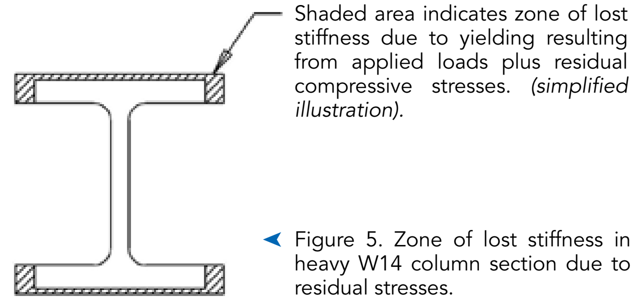

Figure 5. Zone of lost stiffness in heavy W14 column section due to residual stresses.

Loss of stiffness reduces the effective cross-section, which increases column slenderness and reduces column strength. The effects of residual stresses on column strength are accounted for in the Chapter E column strength equations E3-2 and E3-3 and can be seen when the column strength equations are plotted with the Euler column strength equation (see Figure 6).

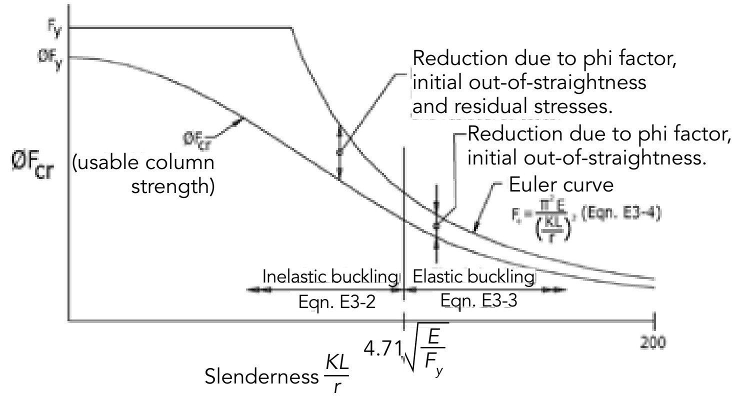

Figure 6. Column strength curve.

Uncertainty in strength and stiffness. Uncertainty in strength and stiffness must be considered in order to provide a margin of safety due to variability in material properties, dimensional tolerances, camber, sweep and other variables. This uncertainty is accounted for in the member strength equations and safety factors.

Second-Order Analysis Procedures

Section C2.1 of the *Specification* requires that P-Δ and P-δ second-order effects always be considered. P-Δ effects are usually the result of lateral loads acting on the structure; P-δ effects are usually the result of column moments due to gravity loads. When moments from lateral loads are significantly larger than those from gravity loads, P-Δ effects will be usually be significantly greater than P-δ effects.

Section C2.1(2) permits P-δ effects to be ignored when all of the following conditions occur:

a) Columns are vertical
b) Ratio of second-order to first-order drift is < 1.7
c) No more than one-third of the total gravity load is supported on moment frame columns

Section C2.1 requires that second-order effects be computed either by performing a rigorous computer analysis or by using an approximate method. One approximate method for computing second-order effects is provided in Appendix 8. That method involves computing moment magnification factors B_1 and B_2 to account for P-δ and P-Δ effects respectively.

The Role of the K-Factor

The effective length factor, "K," is a modification factor applied to the length of columns with defined restraint conditions at each end that is used to determine the lengths of pinned-pinned columns with equivalent flexural buckling strength.

The Euler equation defines the theoretical elastic buckling capacity, P_e of perfectly straight columns, with no imperfections and no residual stresses—unrealistic assumptions for columns in buildings. The Euler equation defining elastic flexural buckling stress is provided in the AISC *Specification* as follows:

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad \text{(E3-4)}$$

Over the years various column strength curves have been developed based on load tests. *Specification* Equations E3-2 and E3-3 (see Figure 6) are the equations used today to predict the flexural buckling strength of steel columns. Equation E3-2 defines flexural buckling capacity for stocky columns in the inelastic zone where yielding occurs within the cross section prior to buckling:

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y \qquad \text{(E3-2)}$$

Equation E3-3 defines column strength for slender columns where buckling occurs prior to yielding within the cross-section:

$$F_{cr} = 0.877\, F_e \qquad \text{(E3-3)}$$

Equations E3-2 and E3-3 (which are based on the Euler equation) were developed to "best fit" the results of load tests on columns with residual stresses and imperfections. Accordingly, equations E3-2 and E3-3 capture the adverse effects of residual stresses and geometric imperfections (P-δ effects) for steel columns that are out-of-straight by up to $L/1000$.

Understanding the basis of equations E3-2 and E3-3 helps designers understand the relationship between column K-factors and second-order stability analysis, as well as why K=1 when stability design is performed using the direct analysis method and why K>1 when the effective length method is used in moment frame structures.

Geometric imperfections and residual stresses have adverse effects on global stability. If a second-order stability analysis considers geometric imperfections and residual stresses, then the effective length factor can be ignored. The direct analysis method considers geometric imperfections and residual stresses in the second-order stability analysis, thus K=1 when the direct analysis method is used.

When the effective length method is used, destabilizing effects of residual stresses are not accounted for in the second-order stability analysis. The destabilizing effects of residual stresses global stability are accounted for in the column strength equations by computing column strength based on the effective length, KL (see Figure 7).

When the direct analysis method is used, destabilizing effects of residual stresses are accounted for by using reduced column stiffnesses in the second-order stability analysis. Computation of the effective length factors to capture the adverse effects of residual stresses on global stability within the column strength equations is not required.

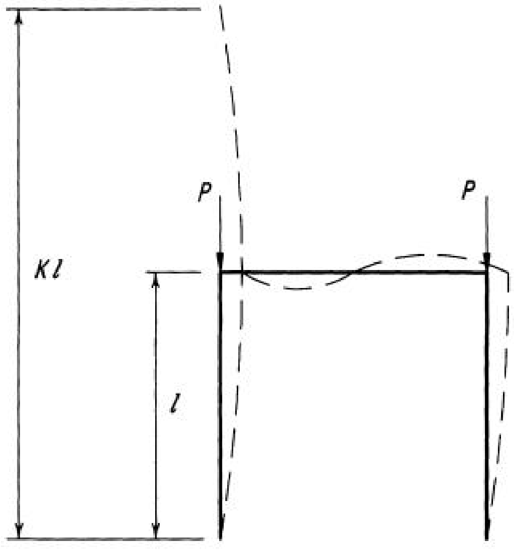

Fig. C-C2.1. Column effective length.

Figure 7. The effective length method accounts for the adverse effect of residual stresses on global stability by using the effective column length, KL, when computing column strength.

The Effective Length Method

A procedure similar to the effective length method was used by practicing engineers for many years prior to development of the direct analysis method. The current effective length method (delineated in Appendix 7) is much more comprehensive and specific than that earlier procedure.

Of the three stability design methods, the effective length method is by far the one for which the stability analysis and the member strength equations play equal roles in investigation of global stability.

First-order effects, second-order effects and initial column out-of-plumbness are considered in the second-order stability analysis.

Adverse effects of residual stresses are not considered in the second-order analysis. They are instead accounted for in the column strength equations using a column length, *KL*. Verification of stability is a two-step process. First, the effects of applied loads and out-of-plumb columns on global stability are analyzed through a second-order stability analysis to determine required member strengths. Second, the effects of residual stresses, geometric imperfections and uncertainty in strength and stiffness on stability are quantified by computing the available column strength with the Chapter E strength equations using column lengths, *KL*.

The use of the effective length method is not permitted for stability-sensitive structures—i.e., structures where the ratio of second-order to first-order effects are greater than 1.5. The upper limit is mandated in part because the adverse effects of residual stresses on stability are not accounted for in the second-order analysis. While relying on the column strength equations to account for the adverse effects of geometric imperfections and residual stresses on global stability is not as exact as considering them directly in the second-order stability analysis, the margin of error is acceptably small for structures that are not stability-sensitive.

Determination of accurate K-factors is required. K-factors determined from the nomographs in Appendix 7 must be adjusted to account for actual conditions, which often differ from the idealized conditions upon which the nomographs were developed.

When structures are stiff enough such that the ratio of second-order drift to first-order drift is less than 1.1, then the columns in moment frames may be designed using K=1. The rationale for using K=1 for stiff structures is that the adverse effects of geometric imperfections and residual stresses on global stability will be sufficiently small when sway frames are very stiff. The use of K=1 for stiff structures essentially modifies the effective length method to a "quasi direct analysis method," where if geometric imperfections and residual stresses were to be considered in the second-order stability analysis, then their effect on stability would be likewise negligible.

There are two limitations to the effective length method. Gravity loads must be supported by vertical columns and the ratio of second-order to first-order effects must be < 1.5

The effective length method procedure is as follows:

1. Perform the analysis at strength level loading (7.2.1(2))
2. Perform second-order analyses (considering both P-Δ and P-δ effects) to determine required member strengths:
   a. Gravity loads + lateral loads
   b. Gravity loads + notional loads (Ni)
3. Compute K-factors for columns providing lateral stability to the structure
   a. Braced frame structures, K=1
   b. Moment frame structures where the ratio of second-order drift to first-order drift <1.1, K=1
   c. Moment frame structures where the ratio of second-order drift to first-order drift >1.1, K>1.
4. Compute strength of columns providing lateral stability to the structure using the Chapter "E" strength equations and using effective lengths, *KL*.
5. Verify that the available column strengths (step #4) are greater than the required strengths (step #2)

Disadvantages of the effective length method:

1. Cannot be used for stability-sensitive structures
2. Requires determination of column K-factors (tedious and time-consuming)
3. Not as direct and intuitive as the direct analysis method

First-Order Analysis Method

The first-order analysis method is a variation of the direct analysis method. This method is not permitted to be used for stability-sensitive structures. The ratio of second-order drift to first-order drift must be < 1.5.

A second-order analysis is not required. Second-order effects are approximated through use of notional loads, the magnitudes of which are proportional to the first-order drift. The notional loads are larger than the ones used in the direct analysis and effective length methods, and they must be included in all load combinations, including those with other lateral loads. The magnitude of the notional loads depends on the magnitude of the first-order drift. Since first-order drift depends on the notional load and the notional load depend on the drift, the easiest way to avoid an iterative procedure is to first establish the maximum permissible drift that can be tolerated (drift under strength level loading), compute the corresponding notional load, perform the first-order analysis and verify that the drift is less than the limit established.

All issues related residual stresses are eliminated by requiring that axial load stresses in sway columns be sufficiently small so that residual stresses will not adversely affect column stiffness.

Limitations of the first-order analysis method are the same as those listed above for the effective length method.

The procedure for the first-order analysis method is as follows:

1. Perform the analysis at strength level loading (7.3.1(2))
2. Size columns in moment frames to limit axial stress, αPr $< 0.5 \times P_y$ (A-7-1)
3. Add an additional notional load to all load combinations, $Ni = 2.1\alpha(\Delta/L)Yi \geq 0.0042\ Yi$ (A-7-2)
4. Perform a first-order analysis to determine required strengths.
5. Compute column strengths using the Chapter "E" strength equations
6. Verify that the available column strengths (step #5) is greater than the required strength (step #4)

Disadvantages of the first-order analysis method:

1. It cannot be used for stability-sensitive structures
2. Limits on column axial stress may require columns larger than with the direct analysis method.
3. It is not as direct and intuitive as the direct analysis method

Direct Analysis Method

The direct analysis method offers advantages over both the effective length and first-order analysis methods.

The direct analysis method is easy-to-understand, intuitive and versatile. There are no limitations on its use because all issues affecting global stability are fully considered and accounted for in the second-order stability analysis.

This method eliminates the need to consider effective length factors and unlike the effective length and the first-order analysis methods, there is no upper limit on the ratio of second-order to first-order drift. However the Commentary recommends that the ratio of second-order to first-order drift not exceed 2.5. Structures where the ratio is exceeds 2.5 can be extremely sensitive to small increases in load or small changes in stiffness. Limiting the ratio to 2.5 ensures that structures will have a sufficient stiffness to prevent runaway instability in the event that there are unanticipated additional loads or minor reductions in member stiffness.

The procedure for direct analysis method is as follows:

1. Perform the analysis at strength level loading (C2.1(4))
2. Apply notional loads, Ni=0.002 αYi at each floor (C2.2b). Apply notional loads for all load combinations, except that they need only be included in gravity-only load combinations when ratio of second-order to first-order drift is < 1.7.
3. Modify stiffness of all members contributing to the lateral stability of the structure
   a. Reduce *axial* and *flexural* stiffness of members contributing to the stability of the structure by a factor of 0.80. (C2.3(1))
   b. Reduce *flexural* stiffness of members contributing to the stability of the structure by an additional factor, τ_b. This factor accounts for additional loss of stiffness due to residual stresses when compression stresses in columns are high. (C2.3(2))
      i. $\tau_b = 1$ when $P_u/P_y \leq 0.5$
      ii. $\tau_b = 4\,[(P_u/P_y) \times (1-(P_u/P_y))]$ when $P_u/P_y > 0.5$
      iii. alternatively, when $P_u/P_y > 0.5$, use $\tau_b = 1$ and apply an *additional* notional load 0.001αYi at each floor per Section C2.3(3)

   The provision to permit the use of $\tau_b = 1.0$ when $P_u/P_y > 0.5$ allows the adverse effects of residual stresses to be conservatively accounted for by increasing second-order P-Δ effects through the use of higher notional loads rather than reducing column stiffness further.
4. Perform a second-order analysis for all load combinations using the reduced stiffnesses and notional loads. The second-order analysis provides the required design strengths for the members in the lateral load resisting system. Consider both P-Δ and P-δ effects. (C2.1(2)).
5. Determine the available strength of all members in the per Chapters C through K
6. Verify that the available column strengths (step #5) are greater than the required strengths (step #4) MSC

This article serves as a preview of Session N3 at NASCC: The Steel Conference, taking place April 17-19 in St. Louis. Learn more about the conference at www.aisc.org/nascc.