Stability Analysis: It's not as Hard as You Think

BY CHRISTOPHER M. HEWITT, S.E.

The Direct Analysis Method is a good choice for stability design—and with a little guidance, it can be a relatively simple process.

STABILITY IS FUNDAMENTAL TO DESIGN, YET IT CAN BE CHALLENGING TO understand, as many of the current provisions are new. The AISC *Specification* allows designers to use any method of stability analysis that considers each of the following:

- Second-order effects
- Flexural, shear, and axial deformations
- Component and connection deformations
- Member stiffness reduction due to residual stresses
- Geometric imperfections

Each of these effects is considered in all three stability design methods presented in the AISC *Specification* (the Effective Length Method in Section C2.2a, the First-Order Analysis Method in Section C2.2b, and the Direct Analysis Method in Appendix 7). The Direct Analysis Method is discussed here, as it offers an advantage: It eliminates the need to calculate *K*-factors in design. The *K*-factor, a long-standing feature of structural frame design, is well accepted as a means to implicitly capture many of these effects, in spite of its many limitations and underlying assumptions that are rarely satisfied in real structures.

The Effective Length Method is still permitted with minor changes in the AISC *Specification* and remains based in the use of *K*-factors in design. But after evaluating the Direct Analysis Method, you may see that it allows for a more transparent, intuitive, and often easier approach to design for stability.

Buckling

The most fundamental theoretical formulation in stability design and buckling of columns is the Euler formula, which defines the elastic axial buckling strength of a member as:

$$P = \frac{\pi^2 EI}{L^2}$$

The theory behind this strength equation assumes that the column is perfectly plumb and straight, behaves elastically, and has pinned ends that are restrained against lateral movement. These assumptions all are commonly violated in real structures. In practice, columns lean due to fabrication and erection tolerances and are out-of-straight between braced points due to mill and fabrication tolerances. Residual stresses are present that cause inelastic behavior. Further, as a structure is loaded, deformations and drift also occur, adding second-order forces and moments. Not only must these effects be accounted for in design, but the fact that column buckling always involves both an axial force and bending effects also needs to be recognized.

Second-Order Effects

When flexure is introduced into an axially loaded member from the axial force acting through the sidesway of a frame and curvature of a member, this is referred to as a second-order effect. The analysis of the structure must be modified to capture the impact of these effects, as they will not be realized in a first-order design model of initially plumb frames and straight members. The primary effects to be considered are P-Δ moments, associated with the sidesway of the structure, and P-δ moments, associated with the curvature of each individual member as it deflects and deforms.

The moments generated by these effects can be captured in the analysis in several ways, and any method that a designer chooses to analyze a structure that captures each of the possible effects is acceptable. Therefore, it is equally permissible to analyze a structure by a direct, rigorous second-order analysis, or to use an approximate method of second-order analysis, such as the one presented in *Specification* Section C2.1b.

In the former case, the frame and member deformations are tracked directly within the analysis software as a part of the analysis. In the latter case, a first-order analysis is made and the resulting forces and moments are amplified using the variables B_1 and B_2 from the AISC *Specification*. B_1 captures the amplification of forces and moments due to the curvature or out-of-straightness of the member (P-δ), and B_2 captures the amplification of forces and moments from the drift of the frame (P-Δ). These effects are illustrated for a single column in Figure 1.

Christopher Hewitt is a project engineer with Forefront Structural Engineers in Chicago. He is a former senior engineer with AISC.

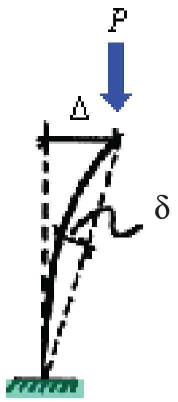

Figure 1. Basic model describing *P*-Δ and *P*-δ effects for a single fixed-base column.

Deformation of the Structure

Engineers are generally familiar with methods of calculating the deflections of members under load. While the structure can be analyzed conventionally for the deflections of individual members, it is important to be sure that these deflections are captured in a second-order analysis of the frame. As stated previously, it is equally permissible to analyze a structure by a direct, rigorous second-order analysis, or to use an approximate method of second-order analysis, such as the one presented in *Specification* Section C2.1b. It also is important to consider the effect of connection and panel-zone deformations in the analysis.

Residual Stresses

Residual stresses are introduced into structural shapes as a result of the production process. Residual stresses include stresses due to temperature, as some elements of the hot rolled cross-section will cool faster than others, and also due to the effects of straightening that must be done to meet ASTM A6 tolerances. Areas with residual stress will yield prior to the overall yielding of the section, causing the column to lose some of its stiffness before reaching its theoretical buckling strength. The effects of residual stresses on member strength are accounted for in the column equations. However, the loss of stiffness due to residual stresses also will increase the frame and member deformations. This is accounted for in the Direct Analysis Method by using a reduced stiffness for all members in the analysis: multiplying the axial stiffness, *EA*, of all members by 0.8 and multiplying the flexural stiffness, *EI*, of all members by $0.8\tau_b$, where τ_b is the column stiffness reduction factor.

Geometric Imperfections

Geometric imperfections are inherent in all structures, and limits on these are found in the AISC *Code of Standard Practice* (plumbness of frames) and the ASTM standards for structural shapes (straightness of members). Frame out-of-plumbness is modeled directly in the Direct Analysis Method using notional loads acting laterally at each floor (alternatively, this can be done by direct modeling of the out-of-plumb frame geometry, if it is known).

A notional load is an equivalent lateral load of appropriate magnitude such that it will generate a story shear in the structural model equivalent to the effect of the axial loads in a story acting in the deformed position, as illustrated in Figure 2. According to the AISC *Code of Standard Practice*, the permissible tolerance on out-of-plumbness of any individual column is no larger than *L*/500, and the notional load is specified to generate a story shear corresponding to this amount of out-of-plumbness. A horizontal notional load of 0.002 times the story gravity load in the horizontal direction is applied, with the 0.002 coefficient being equal to 1/500—the erection tolerance permitted by the *Code of Standard Practice*.

Leaning Columns

In any stability analysis, it is necessary to capture the destabilizing effects of columns that rely on the lateral frame for stability but are not a part of the lateral frame. These columns with pinned ends are commonly referred to as "leaning columns." When modeling the frame, leaning-column effects can be captured either by developing a complete 3D model of the frame or by assigning a single equivalent leaning column carrying the summation of all of the gravity loads on all of the leaning columns in the structure, as a pin-connected part of a 2D frame. An example of how this might be modeled in a 2D analysis is shown in Figure 3.

Step-by-Step Analysis

Now that you know the basics, here is a simple step-by-step process to guide you as you use the Direct Analysis Method:

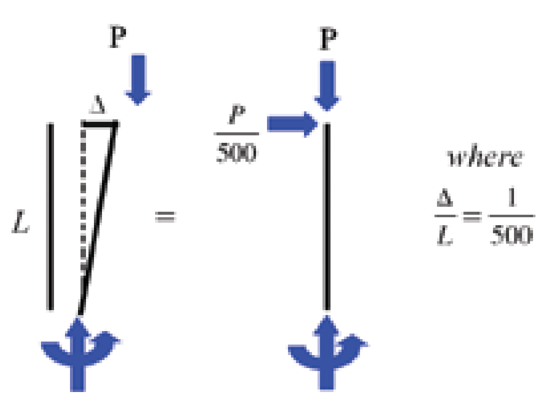

Figure 2. Equivalent loading using notional loads to represent the effect of geometric imperfections on a column.

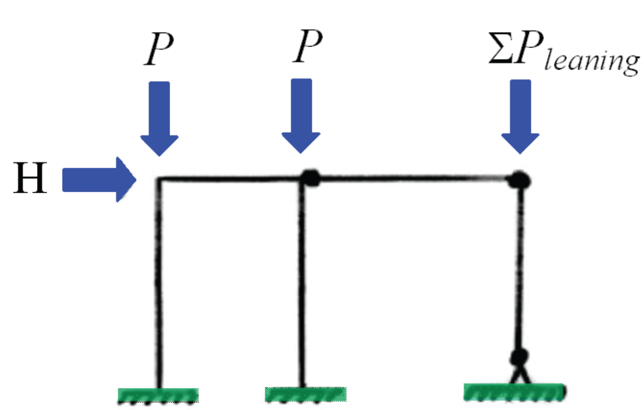

Figure 3. 2D frame model that captures leaning column effects.

1. Create a model of the lateral frame being analyzed, including the leaning columns.
2. Reduce the stiffness of lateral framing members in your model.
   a) **For a braced frame** modeled with pinned connections, this can be done by modifying the modulus of elasticity in your model to 0.8 times *E* (23,200 ksi).
   b) **For a moment frame** (or a braced frame modeled with rigidly connected members), this can be done by modifying the modulus of elasticity in your model to $0.8\tau_b E$. As an alternative to applying τ_b, add a notional load of 0.001 times the gravity load to the notional load in step 3.*
3. Apply notional loads to all load combinations equal to 0.002 times the gravity load† on each story at the story level.**
4. Conduct a second-order analysis of the structure under applied loads, either by the Amplified First-Order Analysis approach (B_1/B_2) or by completing a direct, rigorous second-order analysis of the structure.
5. Reset your modulus of elasticity to E = 29,000 ksi and design the members using the equations in the AISC *Specification* to resist the forces that you have just determined, with $K = 1$.
6. Check drift limits for wind and seismic design.
7. Pat yourself on the back for doing a great job on your analysis! MSC

This topic is discussed in greater detail in a presentation by R. Shankar Nair, S.E., Ph.D.—the 2007 AISC T.R. Higgins Lecture—which can be viewed for free at www.aisc.org/boxedlunch.

Footnotes

* τ_b may be taken as 1 if $\alpha P_r < P_y$. This may, however, require iteration in the analysis if the member sizes change and the inequality reverses. Moment frames will often satisfy this inequality, allowing τ_b to be taken as 1.

† The term "gravity load" refers to either the LRFD gravity load combination or 1.6 times the ASD gravity load combination on each story.

** Per the AISC *Specification*, notional loads only need to be added to load combinations in which the notional load is larger than the lateral load on the frame. Thus, notional loads usually can be ignored in all but the gravity-only load combinations. However, if a designer wishes to simplify the design process, it is always conservative to include the notional loads.