Behavior and Analysis of Curved and Skewed Steel Girder Bridges

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Summary

Curved and skewed steel girder bridges display unique behavior characteristics, some of which are not immediately obvious to the inexperienced designer. In addition to simple vertical flexure behavior there can be significant torsional loading and twisting of the girders. Furthermore, cross-frames begin to act as primary load-carrying members and must be designed as such. If not properly addressed in analysis and design, these effects can result in problems with construction, fatigue or understrength members.

Fully understanding these structures and choosing the appropriate level of analysis are two keys to successful design. Unfortunately, there are no set rules for choosing the level of analysis.

This paper discuses behavior of curved and skewed steel girder bridges, then focuses on analysis techniques. Advantages and disadvantages of each approach will be addressed, including consideration of issues such as required engineering effort, accuracy of the results and appropriate understanding and presentation of the results.

BEHAVIOR AND ANALYSIS OF CURVED AND SKEWED STEEL GIRDER BRIDGES

By Domenic Coletti, P.E., and John Yadlosky, P.E.

Contemporary transportation projects increasingly feature complicated bridges. Specifically, curved and/or skewed steel girder bridges are used on a more frequent basis to solve the challenges of tight geometric constraints associated with constricted urban project sites, complex environmental restrictions and the need to interface with existing infrastructure. The same conditions also add to the severity of curvature and skew.

Fortunately, today's bridge engineers have access to a multitude of tools and techniques to deal with these challenges — arguably, the hardest part of the design process now is choosing the right design method. To this end, designers should be fully aware of the issues associated with design and behavior of curved and skewed steel girder bridges and should completely understand the inherent strengths and weaknesses of the aforementioned tools and techniques when applied to these structures. With this knowledge designers will be able to choose the method that most efficiently leads them to an appropriate design.

Curved steel girder bridges are much more complex than their straight girder counterparts; there are a number of behavior characteristics unique to these structures:

- Global load-shifting behavior
- Twisting and warping effects
- Lateral flange bending effects

The behavior of skewed bridges is likewise more complex than that of non-skewed bridges, and they experience many of the same load and deformation effects as curved bridges even when their girders are tangent (straight).

It is important that bridge engineers address the issues associated with curvature and skew in steel girder bridges in their analysis and design and when presenting information on their plans. Improperly addressing these issues has caused problems with strength, stability and fit-up during construction, leading to construction delays (3), contractor claims and lawsuits. In a worst-case scenario, these problems could result in injuries or deaths.

One way to avoid such situations is educating more bridge designers about the issues associated with curved and skewed steel bridges and the tools and techniques available to address them. Having a complete understanding of the behavior and analysis of curved/skewed steel girder bridges will enable engineers to make better-informed decisions and help alleviate errors and omissions in their plans.

To this end, this paper will review behavior of curved and skewed steel girder bridges and provide detailed discussion of the three levels of analysis available for designing these structures: approximate analysis, two-dimensional (2-D) analysis (also known as grid analysis), and three-dimensional (3-D) analysis.

BEHAVIOR CONSIDERATIONS

The behavior of curved and skewed steel girder bridges can be broadly divided into two categories:

- The Basics — Curved and/or skewed steel girder bridges experience the same effects of gravity loading (dead load and live load) as straight girder bridges. All bridges are subject to shear and bending moment effects as well as vertical deflections and end rotations. These effects are familiar to bridge engineers, so an extensive discussion of these effects is not warranted here. However, it is important to mention them since they are essential components in the total equation of stress and deformation for curved and/or skewed steel girder bridges.
- Curvature and Skew Effects — Torsional stresses, warping and lateral flange bending, load shifting and warping and twisting deformations.

Author’s note: In the following sections, many effects will be characterized as effects of curvature. However, the reader should realize that many of the same twisting and lateral flange bending effects occur in skewed steel girder bridges as will be described in detail later in this paper.

Torsional Stress Effects

In addition to the basic vertical shear and bending effects described above, a curved girder also will be subject to torsional effects. The torsion in curved girders arises from the fact that the center of loading (center of gravity) of each span in a curved girder is offset from a chord line drawn between the supports for that span. This offset represents an eccentricity which, when multiplied by a given vertical load (dead load or live load), results in a torque on the girder (Figure 1).

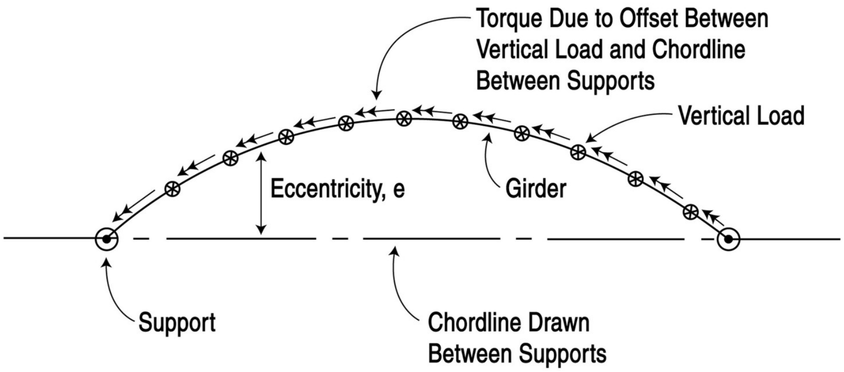
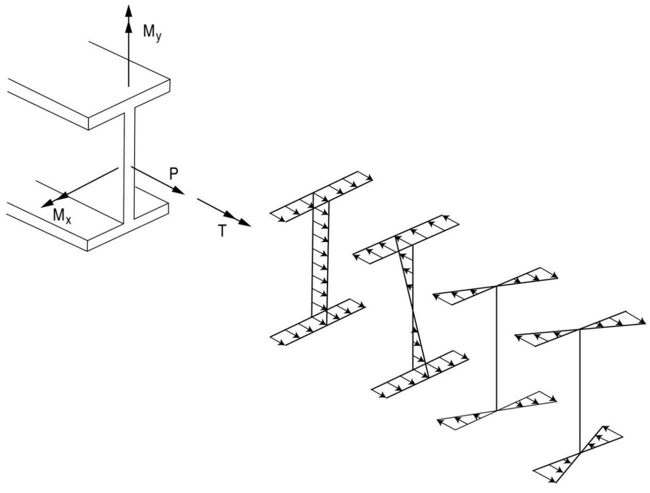


Figure 1: *Plan view of the development of torque in a curved girder. Vertical loads (primarily gravity loads) are applied to the girder at its centerline, but the centerline of a curved girder is not coincident with a straight line (chordline) drawn between support points. The resulting offset represents a moment arm that, when multiplied by the vertical loads, results in a torque on the girder.*



$$\text{Total Normal Stress} = \sigma = \frac{P}{A} + \frac{M_x\, y}{I_x} + \frac{M_y\, x}{I_y} + \text{Warping Normal Stress}$$

Figure 2: *Illustration of the primary normal stresses which can occur in a curved or skewed I-shaped girder.*

Torsion in steel girders causes normal stresses and shear stresses. I-shaped girders and box-shaped girders carry these stresses in different ways, so it is worthwhile to consider them separately.

Because I-shaped girders have low St. Venant torsional stiffness, they carry torsion primarily by means of warping. The total state of normal stress in an I-shaped girder is a combination of any axial stress, primary vertical bending stress, horizontal bending stress and warping normal stress (Figure 2). The total state of shear stress in an I-shaped girder is a combination of vertical shear stress, horizontal shear stress, some small amount of St. Venant torsional shear stress, and warping shear stress (Figure 3).

The relatively low St. Venant torsional stiffness of I-shaped girders is a result of their open cross-section geometry. The St. Venant torsional shear flow around the perimeter of the cross section can only develop force couples across the thickness of any given segment of the cross section. Without a significant force couple distance between these shear flows, the ability of I-shaped girders to carry torque via St. Venant torsional response is low.

Box-shaped girders, on the other hand, are closed-cell structures. Closed cells are extremely efficient at carrying torsion by means of St. Venant torsional shear flow because the shear flow around the circumference of the box has relatively large force couple distances (Figure 4). For this reason, a box-shaped girder can carry

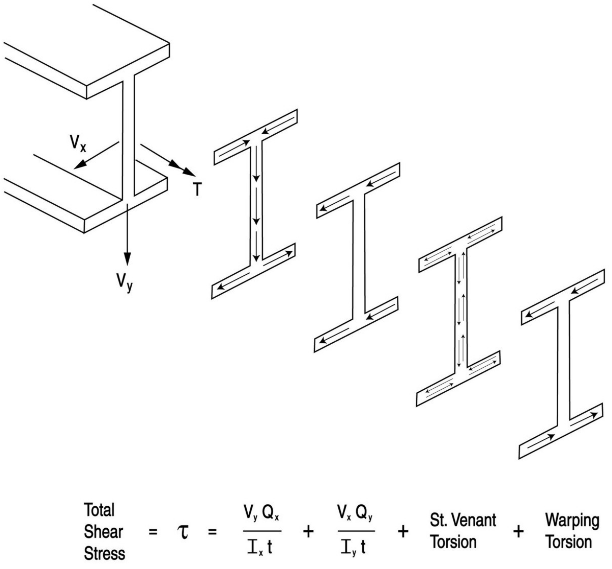


Figure 3: *Illustration of the primary shear stresses that can occur in a curved or skewed I-shaped girder.*

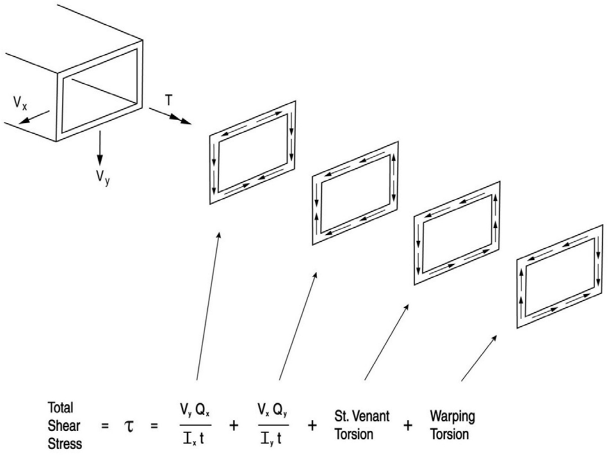


Figure 4: *Illustration of the primary shear stresses that can occur in a curved or skewed box-shaped girder.*

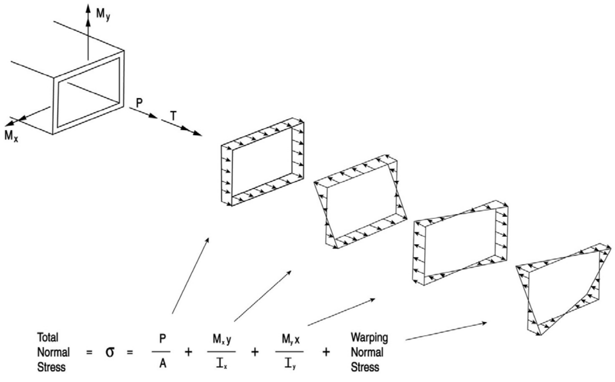


Figure 5: *Illustration of the primary normal stresses that can occur in a curved or skewed box-shaped girder.*

relatively large torques with relatively low shear flows. The shear flow around the circumference of the box follows a consistent direction (clockwise or counterclockwise) at any given location along the length of the girder. As a result, when combined with vertical shear in the webs, this shear flow is always relieving in one web and additive in the other.

As in an I-shaped girder, the total state of normal stress in a box-shaped girder is a combination of any axial stress, primary vertical bending stress, horizontal bending stress and warping normal stress (Figure 5). The total state of shear stress in a box-shaped girder is a combination of vertical shear stress, horizontal shear stress, St. Venant torsional shear stress and warping shear stress (Figure 4).

Lateral Flange Bending

Many practical effects result from the way girders carry torsion. For example, the warping normal stresses for I-girders caused by torsion represent one source of what are called lateral flange bending stresses. These are an important part of the design equations for flange stresses in I-girders. Most curved I-girder analysis techniques include, as a key feature, some method of calculating lateral flange bending stresses, and most formulae for girder design (applied loads/stresses vs. load/stress capacity) include an accounting of lateral flange bending stresses.

It should be pointed out that curvature is not the only source of lateral flange bending stresses. Other causes include wind pressure and seismic events, both of which can induce lateral loads that cause lateral flange bending. Of greater interest to this discussion, though, is the effect of skew in causing lateral flange bending moments. The effects of lateral flange bending in tangent, but skewed, steel girder bridges are often neglected by designers, but it is unconservative to do so. In certain cases these stresses can be significant.

As mentioned previously, skewed bridges exhibit many of the same behaviors as curved girders. For example, in a bridge with straight girders, but with an overall skew and right cross frames, the cross frames will cause lateral flange bending.

Right (non-skewed) cross frames in skewed bridges connect adjacent girders at different positions on the length of each girder, with each girder experiencing different displacement at the point of connection. As a result, these cross frames are subject to forced racking displacements, which cause internal loads in the cross frames (Figure 6). The cross frame loads include horizontal components that induce lateral flange bending effects, very much analogous to the effects that are the basis of the V-Load method of curved girder analysis discussed later.

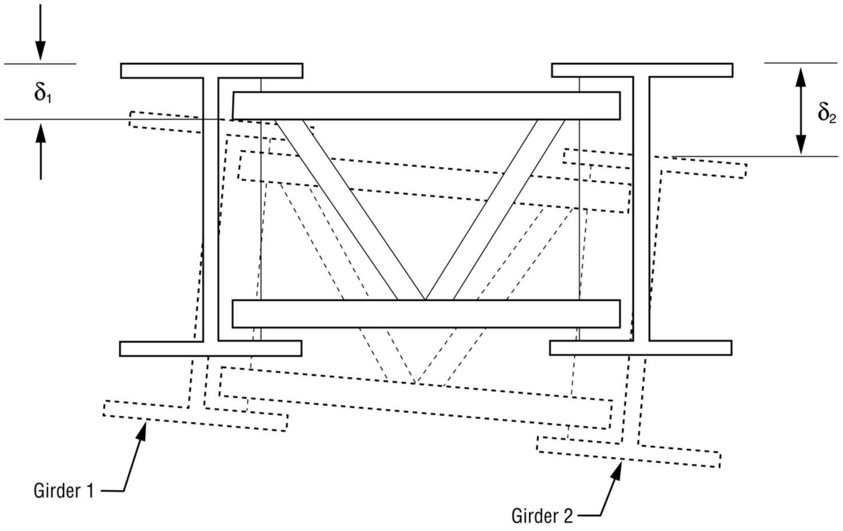


Figure 6: *Right (non-skewed) cross frames in skewed bridges connect girders at different points along their span length. As a result, the cross frames are subject to differential displacements. Due to their high in-plane stiffness, they undergo an in-plane rotation rather than racking. The top corners of the cross frames move horizontally, causing lateral flange bending in the girders.*

Furthermore, near the ends of the girders in skewed bridges, cross frames begin to act as alternate load paths as their stiffness approaches or exceeds that of the girders. Even if select cross frames are oriented on the skew, or if select cross frames are omitted (6), the remainder of the cross frames still undergo this type of behavior and cause the skewed girder system to exhibit many of the same characteristics as a curved girder system, even if the girders themselves are straight.

Many designers believe they can avoid this effect by skewing the cross frames so that girders are not connected at points of differential deflection, but this does not completely eliminate the introduction of cross frame-induced lateral flange bending. Reference 2 touches on this topic, but further discussion is warranted here. Bending rotations (rotations about the transverse axis of the girder) are associated with vertical deflections of the girders caused by primary vertical bending. These are primary bending rotations well-

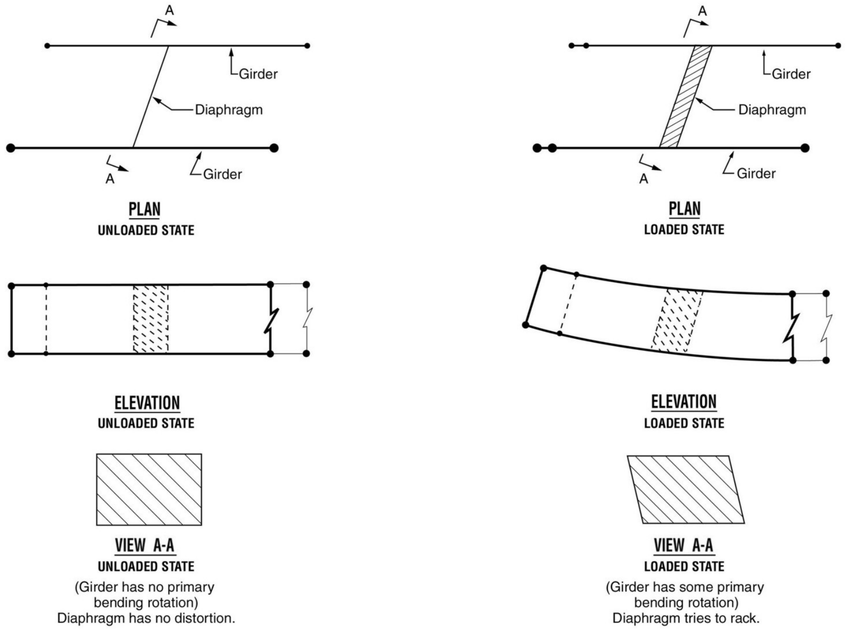


Figure 7: *Cross frame skewed to match the bridge skew also induce lateral flange bending. Girders undergo primary bending rotation as well as deflection, and cross frames must rotate with the girders. But, since the axis of cross frame rotation is not perpendicular to the plane of the girder webs, the cross frames try to rack. However, again due to their high in-plane stiffness they instead experience an in-plane rotation, causing lateral flange bending.*

known to all structural engineers. Assuming uniform bending of all girders in a cross-section, skewed cross frames would connect the girders at points of identical deflection and rotation.

However, as the cross frames rotate to match the primary girder rotations, they also try to rack because they are trying to rotate about the transverse axis of the girders, which is not coincident with the centerline axis of the cross frames since they are skewed. However, due to their high in-plane stiffness, the cross frames experience an in-plane rotation rather than racking. So as the top and bottom corners of the cross frames move forward and backward to follow the primary girder rotation, they also move outward and inward within the plane of the cross frame (Figure 7), inducing lateral flange bending in the girder flanges.

The examples above are just a sample of how a straight bridge with a skew exhibits similar behavior to a curved girder bridge and why it must be designed using many of the same approaches. Numerous references offer good discussions of the effects of curvature and skew in steel girder bridges [e.g., (7), (5), and (2) among others].

Torsional Deformation Effects

In addition to causing significant stresses in both I-shaped and box-shaped girders, torsion also causes significant deformations. Curved girders not only deflect vertically, they also twist. They not only experience end rotations, they also warp (Figure 8). Depending on the severity of the curvature, the length of the spans, the framing of the bridge and the magnitude of the loads, these deformations can become very large, sometimes large enough to be a serious consideration affecting the contractor’s ability to assemble adjacent girders in the field.

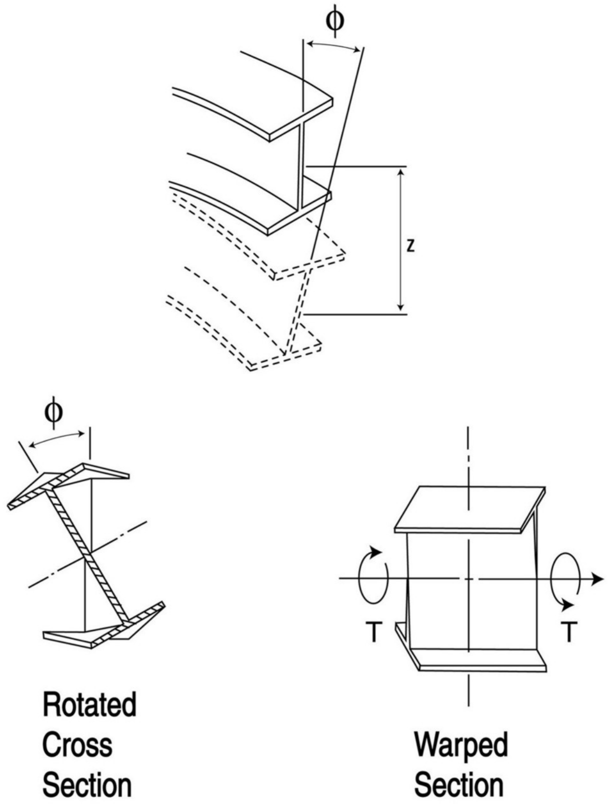


Figure 8: *Illustration of the vertical deflection, twisting deformation and warping deformation experienced by curved steel I-shaped girders.*

Keep in mind also that curved girder bridges are systems, not just collections of individual girders. The sequence of erection, as well as the number of girders in place and connected by cross frames at any given time during erection, will affect their response to loading. Contract plans should clearly indicate the assumed erection sequence and designers should be ready to assess different erection sequences during shop drawing review if the contractor chooses to erect the girders in a different way. The recently released 2005 Interim Revisions to the AASHTO *LRFD Bridge Design Specifications* (1) explicitly require designers to assess these deformations, address them as appropriate on their plans and indicate the assumed erection sequence and intended positions of the girders at various stages of construction. Recent research (8) discusses the magnitudes of these deformations and the ability of various analysis techniques to quantify them.

Again, note that these deformation issues are not exclusively limited to curved girders. Skewed bridges experience many of the same phenomena.

Load Shifting

As was mentioned previously, curved girders experience torsion because their center of loading (center of gravity) is offset from the chord line drawn between their supports (Figure 1). This is equally true for systems of curved girders, in which global overturning causes a load shifting effect whereby girders on the outside of the curve carry different loads than those on the inside.

This effect is similar to how groups of piles carry vertical loads and overturning moments in pile-supported footing. Another analogy is the way bolts carry loads in an eccentrically-loaded bolt group. In all cases, the model used is a rigid-body model in which the applied moment is resolved into force couples that are additive to the primary loads at some points (i.e., additive to the loads in some piles, additive to the loads in some bolts, or additive to the loads in some girders) and relieving at other points. As an example, this behavior in a simple span curved girder bridge results in girders on the outside of the curve carrying more load (Figure 9).

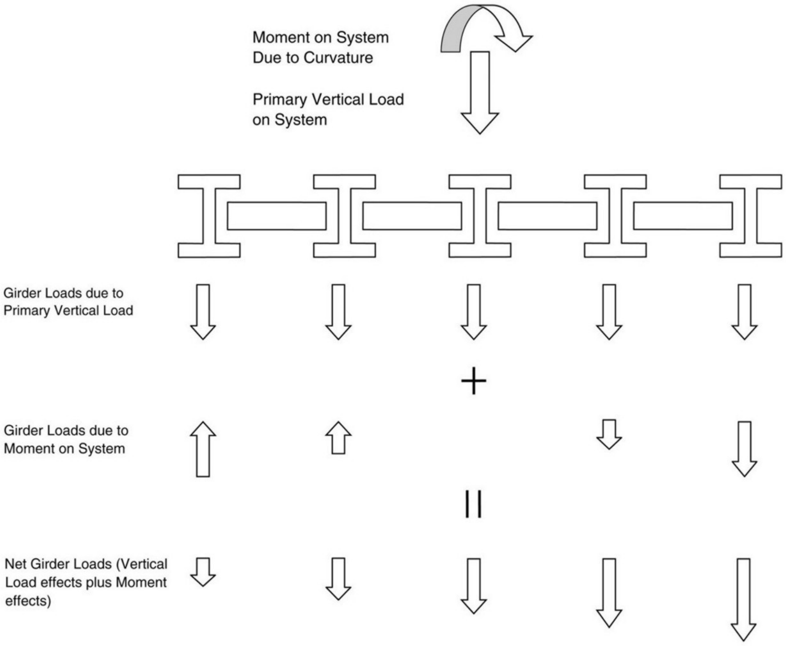


Figure 9: *Illustration of the load shifting phenomenon experienced by curved girders in multiple-girder bridges. The analogy of an eccentrically loaded pile group or bolt group is apparent in this illustration.*

This behavior characteristic generally holds for most curved girder bridges, but designers should be advised to watch carefully for variations in the direction of this type of behavior depending on issues such as the span length balance in multiple span continuous girder bridges (4).

Not only is this load shifting phenomenon itself significant, but the specific load path for effecting this load shifting is also important. Loads are transferred from one girder to the next through the cross frames, which are thus primary load carrying members and must be designed as such.

Behavior Considerations Summary

Through this brief overview of curved and skewed steel girder bridge behavior, we have outlined several key characteristics:

- Torsional shear stresses
- Torsional normal stresses (lateral flange bending)
- Torsional twisting deformations
- Torsional warping deformations
- Load shifting

We also have established that both curvature and skew cause torsional effects in steel girder bridges, including skewed tangent bridges.

LEVELS OF ANALYSIS

Issues

The analysis techniques available for curved and skewed steel girder bridges can be broadly categorized into three groups which, for the purposes of this paper, will be called levels of analysis. The levels are presented in order of increasing rigor of analysis, which corresponds to the order of increasing level of complexity. Each level of analysis has advantages and disadvantages related to accuracy and amount of engineering effort required. Having a good handle on the behavior of curved and skewed steel girder bridges allows designers to better decide when it is appropriate to invest greater time and effort in a more rigorous analysis and more accurately assess the magnitude of the stress and deformation effects resulting from curvature and/or skew in a given bridge.

The simplest techniques fall into the approximate analysis category. These methods typically evaluate girders (as well as other elements, such as cross frames) as independent components, with system effects due to overall geometry and connectivity treated as added forces and moments. Next are 2-D computer modeling techniques, which collectively are called 2-D, grid or grillage analysis techniques. Finally, the most rigorous level of analysis is 3-D analysis. The 2-D and 3-D approaches both analyze the bridge as a full structural system, albeit to different levels of refinement. Each level of analysis will be described in detail, but it would be valuable to first establish some framework for comparing the three levels. For each level of analysis, the paper will address the degree of detail and increased level of refinement of the analysis model.

Generally, the more detailed and exact the analysis, the greater the effort will be to build, run and post-process the analytical model. In addition, an increased amount of output typically is directly associated with increased detail and refinement. While having more output implies having a more detailed picture of the state of stresses and deformations in a structure, it also represents a greater amount of numbers to collate, comprehend and check.

One aspect of bridge analysis common to all three levels of analysis is live load modeling, which deserves special mention as it can be in and of itself a greatly complicating factor in the analysis model.

Live Load Modeling

As all bridge engineers are aware, dead loads on bridges are generally static, unchanging loading conditions. They are fairly simple to address, even considering complications such as addressing the different stages of composite girder construction or the effects of phased construction of bridges. Live loads, on the other hand, are much harder to quantify as they represent a myriad number of loads applied in a nearly infinite number of positions and combinations of positions across the length and width of the bridge. Addressing live load can make the analysis either very simple to perform and understand or make it unmanageable and overwhelming.

There are two primary ways to handle live load modeling for bridge structures. First is what can be called the brute force method, which involves running analyses of multiple live load cases. In computer analysis techniques, this is accomplished using a live load generator — a computer routine that produces literally hundreds or thousands of live load cases, each representing a different load (truck load, lane load, combinations of multiple truck or lane loads, etc.) applied at different positions along the structure. For each live load case, the analysis model is fully calculated and shear and moment results for all key members are developed. The multiple live load case method generates a huge pool of numbers to develop the force envelopes for various members in the structure.

An alternative to the multiple live load case method is the influence line, or influence surface, method. An influence surface is an influence line approach applied in two dimensions rather than just one dimension. A full explanation of the influence surface method is beyond the scope of this paper, but a summary description is warranted.

In this approach to live load modeling, the response of a given point in the model (e.g., a point on a girder, deck, cross frame, etc.) is calculated for all possible positions of a unit load. Instead of presenting these responses in terms of the results of multiple iterative analyses, however, the responses are directly presented in terms of the maximum and minimum response. The influence surface approach to modeling live load effects thus allows the designer to quickly zero in on the maximum loading responses of the structure at given locations. The amount of output from an influence surface analysis is much less, and the designer can focus on the critical loading effects rather than spending substantial time collating thousands or millions of numbers to determine envelope results.

The value of an influence surface approach will become apparent for the more complicated and involved levels of analysis discussed later.

APPROXIMATE ANALYSIS TECHNIQUES

Approximate analysis techniques for curved and skewed steel girder bridges cover a range of methods. The most commonly used methods are described here.

Approximate analysis techniques have several advantages that make them attractive for specific applications, even in this age of computerization. Many of the approximate analysis techniques are based on free-body diagram theories and are quite transparent, giving the engineer a good feel for the distribution of forces through a bridge. Most approximate analysis techniques also are quite simple and quick to use, making them valuable for preliminary designs or as approximate tools for validating more complex analyses.

However, approximate analysis techniques should only be considered rough tools for bridges with anything beyond the most basic geometry and framing. The simplifications and approximations involved in applying the approximate analysis techniques to more complex bridges tend to reduce the accuracy of their results, particularly with regard to the prediction of structural deformations. Use of approximate analysis techniques should be limited to preliminary design or the design of relatively simple structures.

Line Girder Plus Factors

The first approximate analysis technique to be discussed is the line girder plus factors method. Line girder is another term for a tangent (straight) girder. In this method, individual girders initially are analyzed as if they were tangent girders. Any line girder method or tool can be used to determine the shear and moment envelopes for dead load and live load in the girder. For example, the moment distribution method could be used for a totally manual analysis. For a computerized approach, one could use a program such as SIMON (15) or STLBRIDGE (11).

Next, the results calculated by the line girder analysis are increased by factors that account for the effects of curvature (i.e., for the load shifting phenomenon described previously). These factors can be found in several references (e.g., 5).

In addition to addressing the load shifting effects of curvature, designers using this method also must manually account for lateral flange bending effects. This is most easily done using lateral flange bending equations, which again can be found in numerous sources (e.g., 5). The most common expression of this equation is:

$$M_{Lat} = \frac{Md^2}{10Rh}$$

Where:

M_{Lat} = *Lateral Flange Bending Moment*
M = *Girder Primary (Vertical) Bending Moment*
R = *Radius of Curvature*
h = *Depth of the Girder*
d = *Diaphragm (Cross Frame) Spacing*

It should be emphasized that the line girder plus factors method is an approximate method. Its simplicity makes it a valuable tool for preliminary design studies and for performing approximate checks (also known as sanity checks) of more complicated analyses, but it should not be used for final design of curved steel girder bridges.

The V-Load Method

The V-Load method is a technique for analysis of curved steel I-girder bridges originally developed in the 1960s by engineers with Richardson, Gordon, & Associates and United States Steel (7). The V-Load method is based on relatively simple free-body diagrams and static equilibrium equations, is readily learned and understood, and is still popular as it allows the designer to very clearly see how loads are distributed through a bridge.

The V-Load method gets is name from the shears in cross frames, the V-loads in the analysis. The central free-body diagram in the V-Load method is of a cross frame between two adjacent girders. The shear transferred across the cross frame represents the load being shifted from the girder closer to the inside of the curve to the girder closer to the outside of the curve. Completing the free-body diagram is a balancing horizontal force couple directly associated with the lateral flange bending effects found in curved I-girders. Computer tools are available to help automate the V-Load method, including V-Load and VANCK (16).

The V-Load method is not very sophisticated, but offers the advantage of transparency as a simple, statics-based method for assessing force effects in curved girders. While theoretically applicable to complex framing plans (variable girder spacing, flares, bifurcated girders, etc.), using it on such structures is not recommended.

The M/R Method

As mentioned above, the V-Load method is applicable only to I-girders. The corresponding method for tub and box girders is the M/R method developed by Tung and Fountain in 1970 (9). Like the V-Load method, it is an analysis technique derived from simple statics.

The M/R method is most useful for analysis of single tub or box girders, making it applicable for erection analysis of single girders, complete analysis of a narrow bridge with only one tub girder in its cross section, or a single girder as part of a phased-construction plan. Since the M/R method also calculates tub girder rotations, theoretically, it is possible to use it to solve for loads in multiple adjacent girders. But for practical purposes, the calculations for multiple girder systems are too cumbersome unless computer methods are used.

2-D (GRID) ANALYSIS

A 2-D computer analysis — also called a grid analysis or a grillage model — is a modeling method that uses a 2-D grid of nodes to define the structure. Grid models can be built using either finite element modeling techniques and tools or using truly 2-D stiffness modeling methods. Each girder is modeled using a single sequence of line elements. Cross frames are modeled using a single line element per cross frame. The deck is modeled in strips using line elements.

Grid models have the advantage of being fairly simple to build, run and post-process. The level of modeling effort involved in creating a grid analysis is much less than what is typically required for a 3-D analysis. Many engineers find grid models simpler to understand than 3-D models because they find it easier to picture girders as single line elements.

The grid analysis method does have several disadvantages, though. Most of these are related to the simplifications required to describe a 3-D structure in a 2-D model.

Cross frames, whether they are plate diaphragms or truss-type cross frames (K-, X-, or W-frame cross frames) are modeled using a single line element and the structural behavior of what may be a fairly complex truss or plate structure must be approximated as a set of prismatic cross-section properties in a single line element.

The deck — which in an actual bridge is a 2-D plate structure with both in-plane and out-of-plane stiffness characteristics located offset from the neutral axis of the girders — typically is modeled using unidirectional strips. Since the deck plays the predominant role in live load distribution in the actual bridge, some designers question how accurately a simplified model of the deck will reflect the actual live load distribution.

In addition, there is some question as to the ability of a grid model to accurately assess the response of all elements in a given girder (or other structural element) when the girder cross section, regardless of its complexity, is boiled down to just the primary global girder stiffness parameters modeled as a single line element. Grid models have limited capabilities to directly assess localized effects as lateral flange bending and cross-sectional deformations (such as warping deformations). They also have limited capabilities to directly represent the stiffness characteristics of complex cross-sectional shapes such as truss cross frames, lateral bracing, or open-top (quasi-closed) tub girders with a wide bottom flange, two webs, two top flanges, internal intermediate cross frames and top flange lateral bracing.

All of the approximations and simplifications associated with grid analysis models lead to questions about the accuracy of the results, with regard to calculation of internal loads in the structure and deflections, rotations and deformations.

Due to these inherent disadvantages, the grid analysis technique generally is not recommended for the following structures:

- Bridges with more severe curvature and/or skew where deflections, rotations and deformations become more significant
- Bridges with deep girders where the simplifications introduced in reducing the structural properties down to the single line element level may lead to inaccurate results
- Bridges with long span girders where relative stiffness effects become more significant
- Bridges with variable depth girders where the simplifications introduced in reducing the structural properties down to the single line element level may lead to inaccurate results

Computer tools are readily available for grid modeling, including MDX (15) for I-girder and tub girder bridges, DESCUS I (17) for I-girder bridges and DESCUS II (17) for tub girder bridges. Many of these computer tools will build the model, run the model, post-process the results, and do AASHTO code checks on the girders using the model results. Several can perform both AASHTO LFD and LRFD analyses.

In addition, a grid analysis can be performed using any general finite element analysis programs, including commercially available programs such as STAAD (18), SAP2000 (12), GTSTRUDL (13), LARSA (14), etc. Be aware, though, that significant effort might be involved in building and post-processing the analysis model and in performing AASHTO code checks on the girders, cross frames and other elements.

3-D ANALYSIS

Typically, the most analytically complicated level of analysis is 3-D analysis. Similar to some grid analyses, a 3-D analysis is a finite element modeling technique. But instead of limiting the model to a 2-D grid of nodes and line elements, a 3-D analysis models all of the various pieces of the bridge in three dimensions. Girder flanges are modeled using beam or plate elements, webs are modeled using plate elements, and cross frames and bracing are modeled using truss or plate elements (as appropriate for the given cross frame or bracing configuration). The deck typically is modeled using eight-node solid (brick) elements.

How certain features are modeled can greatly affect the results of the analysis, and inappropriate decisions (decisions that are inconsistent with the true behavior of the structure in these areas) can result in erroneous results and an incorrect design. For this reason, there are many critical decisions when building a 3-D analysis model:

- How to model bearings (i.e., the boundary conditions of the 3-D analysis model)
- How to orient the bearings (i.e., which directions of movement are guided, which are restrained, etc.)
- Whether it is necessary to model the substructures (in certain cases, substructure stiffness/flexibility can have significant effects on the behavior of the superstructure)
- How to model structural connections (e.g., cross frame connections to the girders, lateral bracing connections, etc.)

- How to account for offsets between girder flanges to the neutral axes of cross frame and bracing members
- How to account for offsets between the girder flanges to the deck elements
- How to model connectivity of the girders and the deck
- How to model the moving live loads
- How to model staged deck placement
- How to model staged girder erection
- How to account for centrifugal force effects and effects of deck super-elevation
- How to account for girder out-of-plumbness

Since all of the pieces and parts of the bridge are directly modeled, a 3-D analysis has the advantage of being a very rigorous analysis. A 3-D analysis directly models all stiffness characteristics of the bridge, and direct analysis results are available for all elements of the structure. Complex structural configurations are modeled in detail, rather than approximating the overall stiffness parameters with estimated single values. For example, to model a tub girder in a 3-D analysis, the bottom flange is modeled with separate elements, as are the two webs, the two top flanges, the internal cross frames and the top flange lateral bracing. In contrast, in a grid analysis the entire tub girder (flanges, webs, internal cross frames and top flange lateral bracing) is modeled as a single line element, with the stiffnesses of all the associated complex framing approximated using simplified calculations or empirical estimates. This greater detail and rigor in a 3-D model theoretically leads to more accurate analysis results.

However, 3-D analysis involves much greater modeling effort than grid analysis. The resulting model is significantly more complex, and as a result, there is an increased chance that errors may inadvertently be introduced to the analysis. Furthermore, the greater detail and greater volume of direct results for each and every element of the structure can be a two-edged sword. While there is value in having direct results for all elements of the structure, the sheer volume of the results can become overwhelming in terms of the required post-processing effort. Many designers find 3-D analyses results much less intuitive and harder to visualize and understand. And, in the end, a 3-D analysis is only as accurate as the assumptions made in building the model (discussed above). As a result, there is greater risk that mistakes in the analysis will be missed or that analysis results will be misinterpreted. In sum, there is a very real question associated with 3-D analysis: Is the greater accuracy and detail worth the effort?

While there are obvious disadvantages to 3-D analysis, there are virtually no limitations to the type or complexity of structures that can be modeled using this technique. The limitations come down to time and money available to perform the analysis.

Some computer tools are available to perform part or all of a 3-D analysis. The BSDI 3-D System (10) has the capabilities to build, run and post-process a 3-D FEM model, including features that perform AASHTO code checks on the girders and features that summarize the deflections, cross frames forces and bearing reactions. Other commercial programs perform various parts of this process.

In addition, a 3-D analysis can be performed using any general finite element analysis program, including commercially available programs such as STAAD, SAP2000, GTSTRUDL, LARSA, etc. Be aware, though, that a significant amount of effort might be involved in building and post-processing the analysis model and performing AASHTO code checks on the girders, cross frames and other elements — particularly considering 3-D models provide girder results in terms of flange and web stresses or forces, while many AASHTO design equations are written in terms of overall girder moments and shears.

SUMMARY

Curved steel girder bridges are subject to complex global load-shifting, twisting and warping, and lateral flange bending effects. Skewed steel girder bridges experience many of these same phenomena, even if the girders are straight. The behavior characteristics have significant effect on the strength and constructability of a curved or skewed steel girder bridge and should be appropriately quantified as part of the design effort. Modern bridge engineers have a wide range of tools and techniques at their disposal to achieve this goal.

These methods can be broadly categorized into three levels of analysis available for these types of bridges: approximate analysis techniques, 2-D (grid) analysis techniques, and 3-D analysis techniques. In general terms, use of the approximate analysis techniques should be limited only to preliminary design or approximate design of relatively simple structures. Grid analysis is considered the minimum level of analysis appropriate for final design of curved and skewed steel girder bridges. Grid analysis has the advantage of simplicity, less analysis effort, and more intuitively understood results, but the simplifications often required in a grid analysis may lead to inaccuracies in the results. 3-D analysis offers a more rigorous and detailed assessment of the behavior of curved or skewed steel girder bridges, but requires greater analysis effort and results in a more complicated model that might present a greater probability for inadvertent errors and be difficult to understand, check and use.

Unfortunately, there are no hard and fast rules regarding the level of analysis appropriate for a given set of structural or geometric parameters Each bridge is unique, and there are no easy rules that apply to all situations. Instead, the keys to successful design of a curved or skewed steel girder bridge include understanding the behavior of these types of structures, understanding the various analysis techniques available and developing a solid awareness of the advantages and limitations of each approach.

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