

**ADVANCES IN  
DESIGN OF  
NONCOMPOSITE  
LONGITUDINALLY  
STIFFENED STEEL  
BOX SECTION  
MEMBERS FOR  
BRIDGE  
CONSTRUCTION**



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**BIOGRAPHY**

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industrial structures. His research interests are in the areas of structural stability, and behavior and design of steel structures.

Don White is a Professor at the School of Civil and Environmental Engineering at Georgia Tech. Dr. White's research covers a broad area of design and behavior of steel and steel-concrete structures as well as computational mechanics, methods of nonlinear analysis and applications to design. Dr. White is a member of the AISC Technical Committees 4 - Member Design, and 3 - Loads, Analysis and Stability, the AISC Specification Committee, and several NSBA Steel Bridge Collaboration Task Groups. He is past Chair of the SSRC Task Group 29, Second-Order Inelastic Analysis of Frames and currently serves on the Executive Committee of the SSRC.

Charles King works for COWI North America where he is the Senior Steel Specialist. He now works mainly on bridge design but has also designed building structures and aerospace structures. He has worked on the design, verification and construction engineering of various longitudinally stiffened steel structures, including the proposed suspension bridge across the Straits of Messina and the new Canakkale suspension bridge, currently in the design phase. He was a member of European code committees for steel design before moving to Canada and is currently a member of the steel sub-committee for the Canadian bridge design code.

Michael Grubb, PE is a self-employed steel-bridge design consultant with M.A. Grubb & Associates, LLC in Wexford, PA. He has 39 years of experience in steel-bridge design, steel-bridge design specifications, straight and curved steel-bridge research, the development and delivery of training courses on steel-bridge design, and the development of comprehensive steel-bridge design examples.

Anthony Ream, PE, is a Senior Professional Associate with HDR Engineering in their Weirton, WV office. In his 18 years with HDR, Anthony has designed analyzed a wide variety of steel bridges.

## **SUMMARY**

Longitudinally stiffened steel box section members are highly efficient in resisting loads and can be important to the overall economy of steel bridge components such as tower legs, arch ribs and ties, and edge girders. For many of these types of components, limited longitudinal stiffening can provide material savings that justify the additional fabrication cost. For these types of members in larger bridges, longitudinal stiffening is essential to realization of the design. With the exception of rules for bottom flanges of composite tub girders, guidance for design of longitudinally stiffened steel box section members is limited in the current AASHTO LRFD Specifications. A wide range of recommendations and procedures for various aspects of the design of these types of components exists in the literature and in various current and historical standards in the United States

and internationally. This paper summarizes the outcomes of a comprehensive research effort to arrive at state-of-the-art provisions for the design of noncomposite longitudinally stiffened steel box section members in the context of AASHTO LRFD. A major emphasis of this initiative is on conceptual clarity and ease of use, in addition to accurate characterization of the limit states response for a wide range of practical geometries and configurations of these member types. Several of the achievements of this effort also may provide benefits in the design of stiffened bottom flanges of tub girders; however, the major emphasis of this research is on a unified approach to the design of various types of noncomposite longitudinally stiffened (as well as non-longitudinally stiffened) box section members. This paper provides a conceptual overview of the recommended design procedures developed from the above mentioned research, as well as the basis for and background to these recommendations.

# ADVANCES IN DESIGN OF NONCOMPOSITE LONGITUDINALLY STIFFENED STEEL BOX-SECTION MEMBERS FOR BRIDGE CONSTRUCTION

## 1. Introduction

Noncomposite steel box-section members are highly efficient in resisting loads and are used in various important areas of highway bridge construction. The applications include but are not necessarily limited to truss members, arch ribs and ties, rigid-frame members, columns, edge girders, floor beams and steel tower legs (see Figs. 1 through 3). For many of these types of components, longitudinal stiffening can provide material savings that justify the additional fabrication cost. For these types of members in larger bridges, longitudinal stiffening is essential to realization of the design. There exists great potential for improvement of existing methods for calculating the resistance of longitudinally stiffened welded box-section members to achieve gains in the accuracy of their representation of the limit states responses, as well as greater generality and ease of their design application.

Section 2 of this paper provides a conceptual overview of recommended new design procedures, as well as the basis for and background to these recommendations which are explained in detail in White et al. (2018). These recommendations have been developed as a part of a FHWA-sponsored project - IDIQ Task Order 5011. The objectives of this project can be summarized as follows:

- Develop updated and unified AASHTO LRFD provisions for the design of noncomposite steel box-section members (without and with longitudinal stiffeners) subjected to general loading, i.e., axial tension or compression plus biaxial bending, combined with shear due to torsion and flexure;
- Achieve greater consistency between the various box section provisions;
- Extend the accuracy, generality and ease of use of the present LRFD rules; and
- Create a family of design-friendly provisions, conceptually unified & clearly documented and illustrated.

A list of the AASHTO Articles affected by the proposed improvements is provided in Section 3.



**Figure 1:** Lupu Bridge (GSG, 2013).



**Figure 2:** Akashi Kaikyo Bridge (Chou, 2011).



**Figure 3:** Inside view of the 14' tall x 4' wide - transversely and longitudinally stiffened tie girder of the Hoan Bridge, courtesy of F. Russo (Michael Baker International).

## 2. Recommended procedures for design of longitudinally stiffened box-section members

Sections 2.1 through 2.7 address the following aspects of the design of longitudinally stiffened box-section members:

- 1) Calculation of ultimate compressive resistance of longitudinally stiffened plates;
- 2) Calculation of axial compressive resistance of longitudinally stiffened box-section members;
- 3) Calculation of flexural resistance of longitudinally stiffened box-section members;
- 4) Interaction between axial compression, biaxial bending, and shear due to torsion;
- 5) Interaction between axial tension, biaxial bending, and shear due to torsion;
- 6) Additional guidance for arch ribs and ties; and
- 7) Other miscellaneous items.

### 2.1 Ultimate compressive resistance of longitudinally stiffened plates

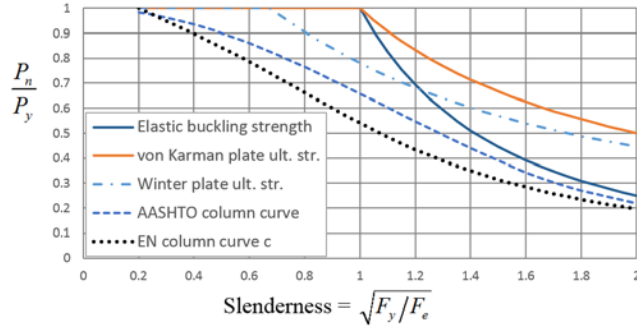
A good prediction of the ultimate compressive resistance of longitudinally stiffened plates is crucial to obtain an accurate characterization of the axial compressive or flexural resistance of longitudinally stiffened steel box-section members. The prediction of the axial compressive resistance of longitudinally stiffened plates involves two steps: 1) Calculating the buckling resistance, 2) Calculating the ultimate compressive resistance. Existing methods for calculating these resistances are summarized in Table 1.

Existing approaches such as Eurocode (CEN, 2006), and AISI (2016) use different combinations and forms of the methods discussed in Table 1 for calculating the buckling and ultimate compressive resistances. The limitations of the various existing approaches are discussed in Lokhande et al. (2018).

The proposed method for calculating the compressive resistance of longitudinally stiffened plates is based on the developments by King (2017) and is explained in detail in White et al. (2018) and Lokhande (2018).

Table 1: Summary of existing methods for calculating the buckling and ultimate compressive resistances of longitudinally stiffened plates

Buckling resistance	Ultimate compressive resistance
<p><b>1) Strut idealization:</b></p> <p>The strut model is based on treating a longitudinally stiffened plate as a series of separate columns comprised of the longitudinal stiffener and an associated plate width.</p> <p><b>2) Column on elastic foundation (CEF) idealization:</b></p> <p>The CEF model considers a longitudinal stiffener (i.e., the longitudinal stiffener and an associated width of the plate) resting on an elastic foundation representing the transverse bending stiffness of the plate. Thus it avoids the limitation of the strut idealization of neglecting the transverse bending stiffness of the plate. This can be significant, especially in relatively narrow plates with one or two longitudinal stiffeners, which are commonly used in North America.</p> <p><b>3) Orthotropic plate idealization:</b></p> <p>The orthotropic plate idealization smears the stiffness characteristics of the longitudinal stiffeners over the entire plate. Thus, it takes into account the longitudinal bending, transverse bending, and torsional stiffness of the plate.</p>	<p><b>1) Column strength curve:</b></p> <p>Mapping to a column strength curve results in no consideration of the plate postbuckling resistance. Column strength curves have a short plateau (see Fig. 4).</p> <p><b>2) Plate strength curve:</b></p> <p>Mapping to a plate strength curve, e.g. Winter's curve, results in a consideration of the plate postbuckling resistance. Plate strength curves have a longer plateau (see Fig. 4).</p> <p><b>3) Interpolation between column and plate strength curves:</b></p> <p>Because of the lack of availability of an explicit ultimate compressive strength curve for longitudinally stiffened plates, the Eurocode (CEN, 2006) requires an interpolation between column and plate ultimate strength curves. Figure 4 clearly shows the higher resistance for plates (Winter's curve) due to the consideration of postbuckling resistance, and also the longer plateau for plate ultimate strengths compared to column strengths.</p>



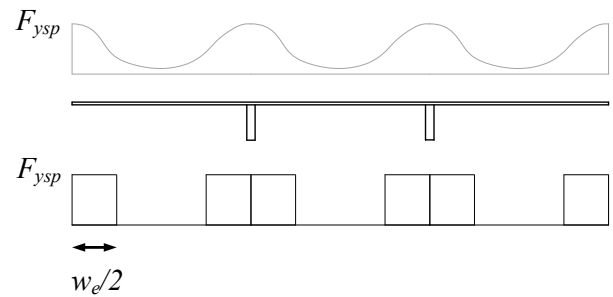
**Figure 4:** Buckling curves

The method is derived using an orthotropic plate idealization and thus considers all three contributions to the buckling resistance - longitudinal and transverse bending stiffness and torsional stiffness. However, the buckling resistance obtained using the orthotropic plate idealization is expressed as an intuitive and easy-to-use column on elastic foundation model, incorporating each of these stiffness contributions. The columns (stiffener struts) are comprised of the longitudinal stiffeners and the associated effective width of the plate. The consideration of the torsional stiffness contributions from the plate gives a longer plateau than that from the column buckling curve alone. As pointed out by King (2017), the torsional stiffness provides much of the stability for plates with a buckling resistance close to the yield stress. Explicit combination of the three contributions to the plate compressive resistance facilitates design optimization since the relative importance of each effect is clear. The flexural buckling resistance of the stiffener struts is quantified using the AISC/ AASHTO column strength curve. Unlike Eurocode (CEN, 2006), the method does not resort to interpolation between column-type and plate-type behavior to determine the extent of the plate-like response.

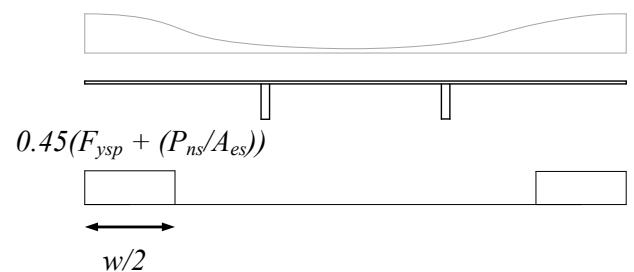
The proposed method is applicable to longitudinally stiffened plates with or without intermediate transverse stiffeners. The characteristic buckling length of the stiffener struts is the theoretical length between the inflection points of their buckling mode for an infinitely long plate. When the spacing between transverse stiffeners and/or diaphragms is smaller than the characteristic buckling length, the buckling length of the stiffener struts is taken as the corresponding spacing. In this case, the strength of the plate is increased due to the transverse stiffening. Otherwise, the spacing of any transverse stiffeners does not impact the resistance of the stiffened plate. The characteristic buckling length of the strut on an elastic foundation is

a familiar concept, and allows the Engineer to make a good decision about the usage and location of any transverse stiffening.

The proposed method recognizes that the stiffened plate edge stress is larger than the ultimate stress of the stiffener struts, and it also takes into account the observation by Lokhande (2018) and King (2017) that the edge stress is typically smaller than the yield stress at the ultimate strength condition. This is captured by performing a linear interpolation between (1) the yield load of the edge,  $P_{yeR}$ , based on the plate effective width tributary to the edge, in the limit that  $P_{ns}$  is equal to  $P_{yes}$ , and (2) the compression force given by a weighted average of yield stress and the maximum compression stress on the adjacent stiffener strut,  $P_{ns}/A_{es}$ , acting on  $A_{gR}$ , in the limit that  $P_{ns}$  becomes small. The force  $P_{ns}$  is the nominal compressive resistance of an individual stiffener strut,  $P_{yes}$  is the effective yield load of the stiffener strut,  $A_{es}$  is the effective area of the strut, and  $A_{gR}$  is the gross area of the laterally-restrained longitudinal edge of the longitudinally stiffened plate element under consideration. The stress distribution and the corresponding effective width for the two extreme situations mentioned above are shown in Figs. 5 and 6.



**Figure 5:** Stress distribution and the corresponding effective width when  $P_{ns}$  is equal to  $P_{yes}$



**Figure 6:** Stress distribution and the corresponding effective width when  $P_{ns}$  becomes small

The proposed method recognizes the postbuckling resistance of the plate panels between the longitudinal stiffeners, and/or between the longitudinal stiffeners and the laterally-restrained longitudinal edges of the stiffened plate, by using an effective tributary plate width when defining the stiffener strut cross-section or edge width.

Certain restrictions are imposed on the longitudinal stiffener cross-sections:

- The slenderness of the cross-section elements of the longitudinal stiffeners should be such that local buckling does not impact the resistance of the longitudinal stiffeners.
- Tee and angle section stiffeners should be sized such that the torsional buckling of these stiffeners about the edge of the stiffener attached to the plate (also known as tripping) is prevented.
- The yield strength of the stiffeners is not to be less than that of the plate to which they are attached.

The proposed method also provides guidance for calculating the compressive resistance of plates with unequally-spaced and/or unequal-size longitudinal stiffeners. These recommendations are an extension of the guidelines for equally-spaced equal-size longitudinal stiffeners described above. They neglect any capability for redistribution of load from weak longitudinal stiffener struts to stronger longitudinal stiffener struts, and provide an accurate to conservative estimate of the compressive resistance.

The proposed method also provides comprehensive and straightforward guidelines for design of transverse stiffeners provided to enhance the compressive strength of longitudinally stiffened plates. The following stiffness and strength requirements must be satisfied by transverse stiffeners to ensure that they are capable of maintaining a node line of negligible deflection in the direction perpendicular to the plane of the stiffened plate:

**Stiffness requirement:** Transverse stiffeners must satisfy the following moment of inertia requirement:

$$I_t \geq 0.05 \frac{P_{up}}{a} \frac{b_{sp}^3}{E} + 0.25 P_{ut} \frac{b_{sp}^2}{E} + 3.8 F_d \frac{b_{sp}^3}{E} \quad (1)$$

where  $a$  is the smallest longitudinal spacing to the adjacent transverse stiffener or diaphragm,  $b_{sp}$  is the total inside width between the plate elements providing

lateral restraint to the longitudinal plate edges, and the other terms are discussed below.

**Strength requirement:** The sum of the axial and bending stresses due to the axial compressive force in the transverse stiffener,  $P_{ut}$ , and the maximum second-order internal moment in the transverse stiffener,  $M_{ut}$ , must be less than or equal to  $\phi_c$  times the yield strength of the plate, where:

$$M_{ut} = P_{up} (\delta_o + \delta) \left( \frac{2b_{sp}}{\pi^2 a} \right) + P_{ut} (\delta_o + \delta) + M_{ut1} \quad (2)$$

$\delta_o$  and  $\delta$  are the transverse stiffener nominal initial out-of-straightness and additional second-order deflection under load, for which equations are provided in the recommended provisions.

In Eqs. 1 and 2,

- The first term addresses the demands due to the longitudinal compression force,  $P_{up}$ , in the stiffened plate.
- The second term addresses the demands due to a concentrically applied internal axial compression force,  $P_{ut}$ , in the transverse stiffener.
- The third term addresses the effects of directly applied loads,  $F_d$ , causing bending of the transverse stiffener.

The second and third terms in Eqs. 1 and 2 are zero in common cases where  $P_{ut} = 0$  and there are no directly applied loads causing bending of the stiffener.

The proposed method imposes restrictions on the transverse stiffener cross-section, which are the same as those specified earlier for longitudinal stiffeners. It however waives the requirement to prevent tripping of the transverse stiffeners if the transverse stiffener, combined with a tributary width of the stiffened plate, is designed as a beam-column or beam member subjected to the axial force,  $P_{ut}$ , and/or the applied bending moment,  $M_{ut}$ . This provides for economical design of these components in large boxes.

The proposed method requires the transverse stiffeners to extend uninterrupted over their specified length. It also requires that transverse stiffeners be attached at their ends to the plate elements providing lateral restraint to the longitudinally stiffened plate edges. In addition, various other longitudinal and transverse stiffener detailing requirements are provided in the recommended provisions.

## 2.2 Axial compressive resistance of longitudinally stiffened box-section members

The proposed method is based on the unified effective area approach (AISC, 2016; AISI, 2016) originally developed by Pekoz (1986). The nominal axial compressive resistance,  $P_n$ , is given as follows:

$$P_n = F_{cr} A_{eff} \quad (3)$$

where:

$$A_{eff} = \sum_{nls} b_e t + \sum_c A_c + \sum_{lsp} (A_{eff})_{sp} \quad (4)$$

in which the summations are over the nonlongitudinally stiffened plates, the corners of the box section, and the longitudinally stiffened plates, respectively;  $b_e$  = effective width of the nonlongitudinally stiffened element under consideration, determined as specified in Article 6.9.4.2.2b of AASHTO (2017) (for non-slender nonlongitudinally stiffened elements,  $b_e = b$ );  $t$  = thickness of the element under consideration;  $A_c$  = area of the box section corner pieces;  $(A_{eff})_{sp} = P_{nsp}/F_{ysp}$  = effective area of the longitudinally stiffened plate under consideration; and  $P_{nsp}$  = nominal compressive resistance of the longitudinally stiffened plate under consideration, calculated using the proposed method in Section 2.1.

$F_{cr}$  = the axial stress on the cross-section effective area at the member nominal compressive resistance, calculated as follows:

If  $\frac{P_{os}}{P_e} \leq 2.25$ , then

$$F_{cr} = \left[ 0.658 \frac{P_{os}}{P_e} \right] F_y \quad (5)$$

Otherwise,

$$F_{cr} = 0.877 P_e / A_g \quad (6)$$

In Equations 5 and 6,

- $P_e$  is the member elastic critical buckling load based on the gross cross-sectional properties.
- $P_{os} = F_y \left( \sum_{nls} b t + \sum_c A_c + \sum_{lsp} (A_{eff})_{sp} \right)$  (7)

Equations 3 through 6 capture the influence of combined local and overall buckling on the member compressive resistance in a simple yet accurate to conservative manner (White et al. 2018, Lokhande 2018).

## 2.3 Flexural resistance of longitudinally stiffened box-section members

Two methods for characterizing the flexural resistance of longitudinally stiffened box-section members are proposed:

**Method 1: Applicable to box-section members with a nonlongitudinally stiffened compression flange and longitudinally stiffened webs.** The characterization of the cross-section flexural resistance of these member types is the same as that proposed by Lokhande and White (2017) for nonlongitudinally stiffened box-section members except the web load-shedding factor,  $R_b$ , is calculated using the load shedding factor for longitudinally stiffened webs (Subramanian and White, 2017), using  $a_{wc}$  determined with  $b_{fc} t_{fc}$  taken as  $A_{eff}/2$  and  $D_c$  taken as  $D_{ce}$  of the effective cross-section, where  $A_{eff}$  is the effective area of the compression flange calculated using the modified Winter's equation given in Lokhande and White (2017).

**Method 2: Applicable to box cross-section members with a stiffened compression flange.** There are three main differences between the proposed method for characterizing the flexural resistance of longitudinally stiffened box-section members with a stiffened compression flange and the method for characterizing the flexural resistance of nonlongitudinally stiffened box-section members in Lokhande and White (2017):

1) Unlike box sections with a nonlongitudinally stiffened compression flange, the effective cross-section of a box section with a longitudinally stiffened compression flange is taken as shown in Fig. 7, where:

$$A_{eff.p2} = P_{nsp} / F_{ysp} \quad (8)$$

$P_{nsp}$  = ultimate compressive resistance of the longitudinally stiffened compression flange determined using the method in Section 2.1;

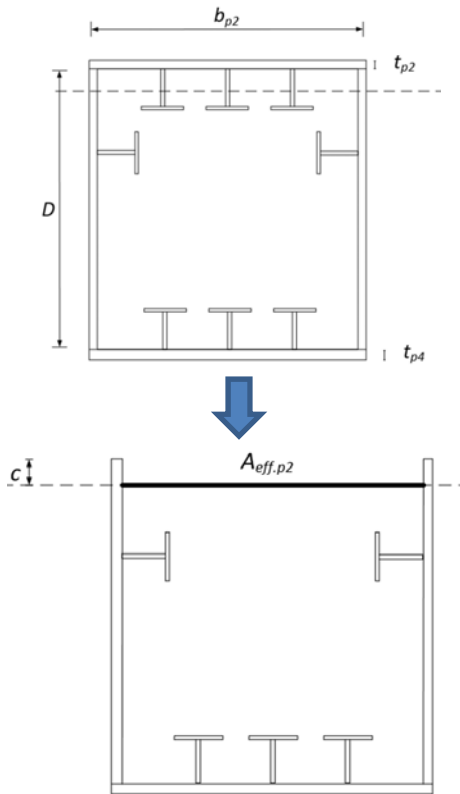
$c$  = the distance of the centroid of the stiffened flange plate from the top of the web plates.

AISI (2016) uses a similar approach in which the effective plate is located at the centroid of the longitudinally stiffened flange plate.

2) Unlike nonlongitudinally stiffened flange plates, longitudinally stiffened flange plates are unable to sustain large inelastic axial compressive strains beyond their maximum resistance. Thus, the flexural re-

sistance of box sections with a longitudinally stiffened compression flange is limited to the first yield of the compression flange in the effective cross-section.

3) Box-section members with a longitudinally stiffened compression flange are limited, conservatively, to the resistance corresponding to the nominal onset of yielding at their tension flange. Research studies show that the resistance of singly-symmetric box sections that have a neutral axis of the effective section closer to the compression flange, and thus are subject to early tension flange yielding, can be calculated with better accuracy by determining the yield moment to the compression flange,  $M_{yce}$ , accounting for the early spread of yielding through the cross-section depth within the tension zone (Lokhande et al. 2018; Lokhande, 2018; White et al., 2018). If the cross-section resistance is calculated in this way, no other impact of early tension flange yielding need be considered. However, the complexity of this calculation is considered to outweigh its benefits. By limiting the maximum flexural resistance to the yield moment to the tension flange, all of the strength limit states are predicted accurately to conservatively.



**Figure 7:** Effective box cross-section considering the resistance of a stiffened compression flange

Salient features of the proposed methods for calculating the flexural resistance of longitudinally stiffened box-section members are as follows:

- The proposed methods account for:
  - The different failure modes of a longitudinally stiffened compression flange plate. The methods do this by more accurately quantifying the flange ultimate compressive resistance (see Section 2.1), which is then used to obtain an effective cross-section.
  - Web bend buckling and the corresponding postbuckling resistance, via the use of the  $R_b$  factor; this avoids the need to perform iterative or two-step calculations when obtaining the effective cross-section.
  - Lateral torsional buckling, including interaction with flange and web local buckling and postbuckling response.
- The proposed methods cover all ranges of component plate slenderness.
- The proposed methods address both singly and doubly symmetric box-section members. In bridges, it is common that fabricated boxes may be singly symmetric; boxes with longitudinally stiffened compression flanges are inherently singly symmetric.
- The proposed methods address box sections with hybrid webs. It is possible for steel box-section members subjected to flexure to have webs with a lower yield strength than one or both flanges.
- The proposed methods recognize the inability of longitudinally stiffened flange plates to sustain large inelastic axial compressive strains beyond their maximum resistance, and therefore, they limit the flexural resistance of box sections with a longitudinally stiffened compression flange to the first yield of the compression flange of the effective cross-section.
- The proposed methods are conceptually consistent with the method proposed by Lokhande and White (2017) for determining the flexural resistance of nonlongitudinally stiffened box-section members.



## 2.4 Interaction between axial compression, bi-axial bending, and shear due to torsion

It is proposed that for members in which all of the cross-section elements are defined as compact for flexure, and are nonlongitudinally stiffened, Eqs. 9 and 10 must be satisfied:

$$\text{If } \frac{P_u}{P_r} < 0.2, \text{ then } \frac{P_u}{2P_r} + \left( \frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}} \right) \leq 1.0 \quad (9)$$

$$\text{If } \frac{P_u}{P_r} \geq 0.2, \text{ then } \frac{P_u}{P_r} + \frac{8}{9} \left( \frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}} \right) \leq 1.0 \quad (10)$$

where  $P_r$  is the axial compressive resistance, and  $M_{rx}$  and  $M_{ry}$  are the flexural resistances excluding tension flange rupture.

These equations, which are the current interaction equations given in AASHTO (2017), provide an accurate to conservative approximation of the resistances under combined loading for these member types. Such members are potentially able to develop significant distributed yielding within their cross-sections for small axial load and dominant flexural loading. As such, these member types are able to develop a “knee” in the interaction curve between their flexural and axial compressive resistances. Members with cross-section elements not defined as compact in flexure generally have a limited capability to develop such a “knee.”

For other box-section members, when the limit state of tension flange yielding (TFY) is defined, the following relationships are proposed:

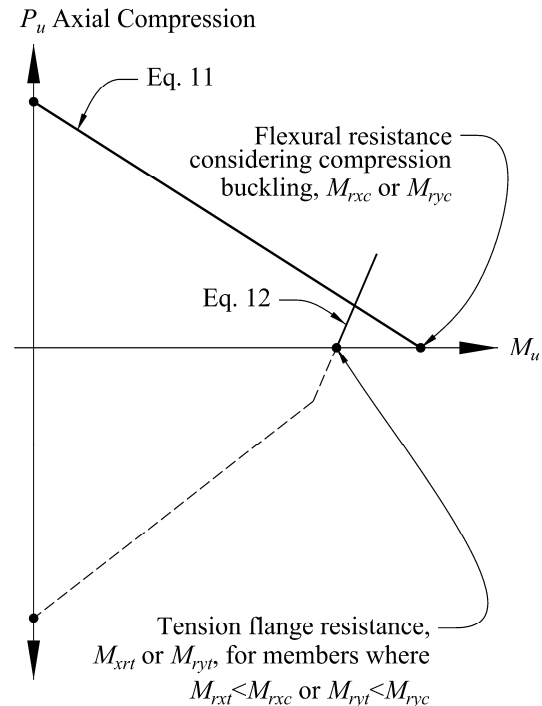
$$\frac{P_u}{P_r} + \frac{M_{ux}}{M_{rxc}} + \frac{M_{uy}}{M_{ryc}} \leq 1.0 \quad (11)$$

$$\left( \frac{M_{ux}}{M_{rxt}} + \frac{M_{uy}}{M_{ryt}} \right) - \frac{P_u}{2P_{ry}} \leq 1.0 \quad (12)$$

Equations 11 and 12 are applicable when the member flange subjected to flexural tension may exhibit early yielding prior to reaching the compression buckling flexural limit states, and where the members generally are not capable of developing extensive yielding in compression prior to reaching their ultimate strength. For these member types, axial compression relieves the influence of early tension flange yielding. This beneficial subtractive effect is captured by Eq. 12. The subtractive axial strength ratio term captures the strength interaction curve form shown in Figure 8. In

Eqs. 11 and 12,  $M_{rxt}$  and  $M_{ryt}$  are the flexural resistances considering only tension flange yielding,  $M_{rxc}$  and  $M_{ryc}$  are the flexural resistances considering compression buckling, and  $P_r$  is the axial compressive resistance, and  $P_{ry}$  is the axial resistance for general tension yielding.

The coefficient of  $\frac{1}{2}$  on the axial strength ratio in Eq. 12 is a conservative representation of the subtractive strength interaction effects. When employing this equation, the Engineer must estimate and use the smallest factored axial compressive force that is applied concurrently with the moments  $M_{ux}$  and/or  $M_{uy}$ . Applying the maximum factored axial compressive force in this equation overestimates the beneficial effects of axial compression on the TFY limit state in flexure, and is therefore unconservative.



**Figure 8:** Interaction between axial force and moment for members where the flexural resistance considering TFY is smaller than the compression buckling flexural resistance

The axial and flexural compression effects are additive here when considering compression buckling. This additive effect is captured by Eq. 11. For members where Eqs. 9 and 10 apply, the member ultimate resistances tend to involve extensive yielding in both tension and compression. In these cases, the above subtractive interaction effects are not realized; that is, all the interaction effects tend to be additive.

The moments,  $M_{ux}$  and  $M_{uy}$ , are to be determined by:

- a second-order elastic analysis that accounts for the magnification of moment caused by the factored axial load, or
- the approximate single-step method specified in AASHTO Article 4.5.3.2.2b, or a comparable amplification factor based procedure.

$P_u$ ,  $M_{ux}$ , and  $M_{uy}$  are the factored axial and flexural forces. The maximum axial force, and the separate maximum member moments about each of the cross-section principal axis,  $x$  and  $y$ , including the second-order effects and irrespective of their location along the member unbraced length, are to be combined together in the applicable equations.

When additive with the corresponding moments  $M_{ux}$  and  $M_{uy}$ , additional bending moments due to the eccentric bending caused by loss of effectiveness of cross-section elements due to local buckling in members with singly-symmetric cross-sections containing longitudinally stiffened plate elements, or elements that are slender under uniform axial compression, can have an impact on the resistance of members having little restraint for their end rotations; however, these effects tend to be minor in members where the end rotations are restrained due to support conditions or continuity with other framing. The recommended provisions suggest that these damping effects are important only when there is small end restraint, consistent with AISI (2012).

Furthermore, it is proposed that when a member is subjected to torsion resulting in factored torsional shear stresses,  $f_{ve}$ , greater than  $0.2\phi_T F_{ver}$ ,

- $P_r$  and  $P_{ry}$  are to be multiplied by  $\Delta$ ,
- $M_{rx}$ ,  $M_{rxc}$  and  $M_{rxl}$  is to be multiplied by  $\Delta_x$ , and
- $M_{ry}$ ,  $M_{ryc}$  and  $M_{ryl}$  is to be multiplied by  $\Delta_y$

in Eqs. 9 through 12, where  $\Delta$ ,  $\Delta_x$ , and  $\Delta_y$  are to be computed as follows:

$$\Delta = 1 - \left( \frac{f_{ve}}{\phi_T F_{ver}} \right)^2 > 0 \quad (13)$$

$$\Delta_x = 1 - \left( \frac{f_{vex}}{\phi_T F_{verx}} \right)^2 > 0 \quad (14)$$

$$\Delta_y = 1 - \left( \frac{f_{vey}}{\phi_T F_{very}} \right)^2 > 0 \quad (15)$$

Equations 13 through 15 address the influence of applied torsion on the resistance of noncomposite box-section members. These equations are based on an interaction between the shear resistance and the combined axial and flexural resistance of the component plates in which the axial and flexural strength ratios are taken as linear terms, with an exponent of 1, and the torsional shear strength ratio is taken as a quadratic term with an exponent of 2. The interaction with the torsional shear is applied to the axial compressive and flexural resistance terms, rather than writing a separate term involving the torsional shear in the strength interaction equations. When  $f_{ve}/\phi_T F_{ver}$  is smaller than 0.2, the reduction in the plate axial compressive resistance due to the torsional shear stress is less than 4 percent, and is therefore neglected.

The interaction assumed by Eqs. 13 through 15 gives an accurate to moderately conservative representation of the plastic strength interaction between normal and shear stresses obtained from the von Mises yield criterion, the inelastic buckling interaction in plates subjected to combined uniform axial compression and shear, and the elastic buckling interaction in plates subjected to combined bending within the plane of the plate and shear (Ziemian, 2010). The theoretical interaction curve between the normalized strength ratios is circular for each of these cases, which would result in the expressions within Eqs. 13 through 15 being taken to the  $1/2$ , or square root, power. However, the plate elastic buckling interaction between uniform axial compression and shear is approximated more closely by an interaction equation involving a linear term for the axial compressive strength ratio and a quadratic term for the shear strength ratio, which results in the form given by Eqs. 13 through 15 (Ziemian, 2010). The largest difference between the overall strengths predicted by the circular interaction and the interaction using a linear term for the axial compressive strength ratio and a quadratic term for the shear strength ratio is 15 percent, corresponding to a shear strength ratio of 0.707. In the proposed approach, the interaction based on using a linear term for the axial compressive and flexural strength ratios is adopted to characterize the member for all of the types of loading considered. This is consistent with the form of the interaction equations given in AISC (2016) for torsion combined with axial force and flexure on hollow structural sections.

The interaction between the flexural shear and the combined member axial and flexural resistances is

taken to be negligible in the above equations. This is consistent with the neglect of moment-flexural shear strength interaction for I- and box-section members within the AASHTO Specifications, as well as within the AISC (2016) Specification.

## 2.5 Interaction between axial tension, biaxial bending, and shear due to torsion

The handling of interaction effects in box-section members subjected to uniform axial tension is similar to the approach discussed in Section 2.4. The relationships in Eqs. 16 and 17 are recommended for all types of members:

If  $\frac{P_u}{P_{ry}} < 0.2$ , then

$$\frac{P_u}{2P_{ry}} + \left( \frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}} \right) \leq 1.0 \quad (16)$$

If  $\frac{P_u}{P_{ry}} \geq 0.2$ , then

$$\frac{P_u}{P_{ry}} + \frac{8}{9} \left( \frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}} \right) \leq 1.0 \quad (17)$$

For box-section members in which the limit state of tension flange yielding is defined, the relationships given in Eqs. 18 through 21 may alternatively be employed, as applicable:

If  $\frac{P_u}{P_{ry}} < 0.2$ , then

$$\frac{P_u}{2P_{ry}} + \left( \frac{M_{ux}}{M_{rxl}} + \frac{M_{uy}}{M_{ryl}} \right) \leq 1.0 \quad (18)$$

$$\left( \frac{M_{ux}}{M_{rxc}} + \frac{M_{uy}}{M_{ryc}} \right) - \frac{P_u}{2P_{ry}} \leq 1.0 \quad (19)$$

If  $\frac{P_u}{P_{ry}} \geq 0.2$ , then

$$\frac{P_u}{P_{ry}} + \frac{8}{9} \left( \frac{M_{ux}}{M_{rxl}} + \frac{M_{uy}}{M_{ryl}} \right) \leq 1.0 \quad (20)$$

$$\left( \frac{M_{ux}}{M_{rxc}} + \frac{M_{uy}}{M_{ryc}} \right) - \frac{P_u}{2P_{ry}} \leq 1.0 \quad (21)$$

It is further proposed that the effect of any torsional shear stresses be considered in a manner similar to that discussed in Section 2.4 via the application of the  $\Delta$ ,  $\Delta_x$  and  $\Delta_y$  terms.

The strength interaction between flexure and axial tension or compression pertaining to tension flange rupture at a cross-section containing holes in the tension flange may be addressed as follows:

$$\frac{P_u}{P_r} + \frac{M_u}{M_r} \leq 1.0 \quad (22)$$

where:

- $M_r$  is the factored tension rupture flexural resistance about the axis of bending under consideration, calculated as

$$M_r = 0.84 \left( \frac{A_{nf}}{A_{gf}} \right) F_u S_t \leq F_{yt} S_t \quad (23)$$

where  $A_{nf}$  is the net area of the tension flange at the bolt holes,  $A_{gf}$  is the gross area of the tension flange, and  $S_t$  is the elastic section modulus about the axis of bending to the tension flange.

- $P_r$  is the factored tensile rupture resistance of the net section at the bolt holes.

Equation 22 focuses on the specific axial force, tension or compression, combined with the specific moment causing flexural tension in the flange at the cross-section under consideration. The axial strength ratio term is negative in Eq. 22, causing a beneficial subtractive effect, when the cross-section having the bolt holes is subjected to axial compression. The axial strength ratio term is positive, causing an additive effect, when the cross-section is subjected to axial tension. This equation is adopted from Section H4 of AISC (2016).

## 2.6 Additional considerations for arch ribs and ties

The action of the axial force in flange longitudinal stiffeners acting through the vertical curvature of an arch rib induces significant radial forces from the longitudinal stiffeners, which much be transferred by the flange plate to the webs of the arch rib. In addition,

for other than potentially free-standing arches, it is unlikely that the flanges of arch ribs would need to be wide enough to benefit from longitudinal stiffening of the flanges. Therefore, the proposed method recommends that longitudinal stiffeners should not be employed for flanges of arch ribs. Guidance for handling these unusual cases is provided in the commentary.

Tee or angle-section stiffeners on the webs of arch ribs would also tend to exhibit significant bending in the direction normal to the stem or leg attached to the web, due to the stiffener axial force acting through the vertical curvature of the arch rib. Therefore, the proposed method suggests that flat plates be used for web longitudinal stiffeners on arch ribs.

It is proposed that a reduced yield strength be used for the calculation of the axial and flexural resistances of arch ribs to account for the influence of transverse plate bending of the flange and web longitudinal stiffener plates due to the axial force in these components acting through the vertical curvature of the arch rib. The proposed reduced yield strength equations are based on recommendations by King and Brown (2001) considering the influence of the transverse plate bending stresses on yielding under the longitudinal normal stresses via the application of the von Mises yield criterion. The transverse bending stresses in the flange plates reduce the yield strength of the flanges on one surface of the plate, and increase the yield strength of the flanges to a lesser extent on the opposite surface of the plate. The proposed equations take the reduced specified minimum yield strength as the average of these corresponding modified yield strengths, multiplied by 1.05, to allow for the conservatism due to the variation of the transverse bending stresses throughout the width and thickness of the plates (White et al., 2018).

Most practical cases do not require any reduction in the specified minimum yield strengths. For a width-to-thickness ratio of 40, considering the clear inside width of a flange between the webs of a box section, the proposed equation gives a value smaller than  $F_y$  when  $b/R$  becomes larger than approximately 0.012, which is larger than the  $b/R$  value for most arch ribs ( $R$  is the radius of curvature of the arch rib at the mid-depth of the web and  $b$  is the unsupported width of the cross-section component plate). Similarly, for a width-to-thickness ratio of 12, considering the projecting width of flange extensions on a box section, or the projecting width of any longitudinal stiffeners, the

proposed equation gives a value smaller than  $F_y$  when  $b/R$  becomes larger than approximately 0.010.

Traditionally, width-to-thickness ratios up to 12 have been permitted for web longitudinal stiffeners in box section arch ribs. For cases where the width-to-thickness ratio of these elements exceeds the stiffener cross-section requirements described in Section 2.1, the axial and flexural resistances of the box-section member may be determined by neglecting the portion of the longitudinal stiffener widths larger than specified by the applicable requirements. If the requirements specified by AASHTO Article 6.10.11.3 are not satisfied for longitudinally stiffened webs, the flexural resistance of the box-section member should be calculated neglecting the longitudinal stiffeners in the calculation of  $R_b$  in AASHTO Article 6.10.1.10.2.

#### Web slenderness limits

To ensure that the webs of arch ribs are sufficiently stout such that they are capable of resisting transverse compression forces developed by the flange compression acting through the vertical curve of the arch rib, it is proposed that the slenderness of the webs of arch ribs should not exceed 90 and should also satisfy:

$$\frac{D}{t_w} \leq 0.40 \frac{\frac{E}{F_{yc}} \sqrt{\frac{A_w}{A_{fc}}}}{\sqrt{1 + \frac{1}{5.6} \frac{D}{R} \frac{E}{F_{yc}}}} \quad (24)$$

This equation is adapted from the Eurocode Part 1-5 (CEN, 2006) equations for the limit state of “flange induced buckling,” traditionally referred to in American structural engineering practice as flange vertical buckling. In the limit that  $R$  approaches infinity and  $A_w/A_{fc} = 1.0$ , Eq. 24 gives the same limit as Eq. F13-4 of the AISC (2016) Specification. Equation 24 tends to govern only for highly curved arch ribs having a small  $A_w/A_{fc}$  and where  $D/t_w$  approaches the maximum recommended limit of 90. For  $F_{yc} = 50$  ksi and  $A_w/A_{fc} = 1.0$ , Eq. 24 gives a limit on  $D/t_w$  less than 90 when  $D/R$  is larger than approximately 0.05. For  $F_{yc} = 70$  ksi and  $A_w/A_{fc} = 0.5$ , Eq. 24 gives a limit on  $D/t_w$  less than 90 when  $D/R$  is larger than 0.01.

#### Flange slenderness limits

It is proposed that the slenderness of any flange extensions satisfy

$$b/t_f \leq 0.38 \sqrt{\frac{E}{F_y}} \quad (25)$$

Traditionally, width-to-thickness ratios up to 12 have been permitted for flange extensions in box-section arch ribs. For cases where the width-to-thickness ratio of these elements exceeds the requirement given by Eq. 25, the axial and flexural resistances of the box-section member may be determined by neglecting the portion of the flange extensions larger than specified by this requirement.

In addition, it is recommended that the width-to-thickness ratio of arch rib flanges should not exceed 40 for the portion of the box section within the clear width between the insides of the webs.

#### Influence of vertical curvature on arch rib LTB resistance

Arch ribs subjected to bending moment causing compression on the flange toward the outside of the vertical curve are less stable with respect to lateral torsional buckling than corresponding straight members with the same cross-section. The recommended provisions provide a simple quantification of this strength reduction.

### 2.7 Other miscellaneous items

The recommended provisions also address the following miscellaneous items (White et al. 2018):

- Service and fatigue limit states and constructibility. Since plate postbuckling resistance is often assumed at the strength limit state in computing the nominal flexural and axial compressive resistance of box-section members with cross-sections composed of slender elements and/or longitudinally stiffened plate elements, the recommended provisions adopt and extend a philosophy from the AASHTO (2017) provisions of ensuring no theoretical local buckling of the component elements at the service and fatigue limit states, and for constructibility.
- Advanced analysis methods. For certain types of steel structures, benefits may be gained by applying advanced analysis methods for the design of the structure and/or its components. Using these methods, the member and structure stability are assessed using a second-order analysis directly considering geometric imperfections and residual

stress effects. These methods provide greater rigor for consideration of innovative structural systems and member geometries that fall outside of routine systems and geometries, which are the primary focus of AASHTO Section 6, as well as capabilities for recognizing additional reserve capacities not addressed by the AASHTO provisions. Using these procedures, no individual checks of member stability are required; the members are checked for their local “cross-section level” resistance. These types of capabilities would typically be applied in design focusing on a limited subset of potentially critical factored design load combinations. Hendy and Murphy (2007) discuss the application of these types of methods in the context of steel bridge design according to the Eurocodes.

- Guidance regarding the use and design of diaphragms in box-section members. This guidance is largely adopted from current guidance in AASHTO (2017) for design of composite box girders.

### 3. AASHTO articles affected by the proposed improvements

Pending approval by AASHTO, the recommendations discussed in this paper and in White et al. (2018) will lead to improvements to the following AASHTO articles:

- 6.9.4.1 and 2 - Nominal Compressive Resistance and Effects of Local Buckling on Compressive Resistance
- 6.7.4.3 and 4 - Diaphragms and Cross-Frames, Composite Box-Section Members and Noncomposite Box-Section Members
- 6.9.4.5 - Service and Fatigue Limit States and Constructibility for Members Composed of Slender Elements and/or Longitudinally Stiffened Plate Elements
- 6.12.1, 6.12.2 and 6.12.2.2 - Miscellaneous Flexural Members, General Nominal Flexural Resistance, and Noncomposite Rectangular Box-Section Members
- 6.9.2.2- Combined Axial Compression, Flexure and Torsion

- 6.8.2.3- Combined Axial Tension, Flexure and Torsion
- 6.14.4- Solid Web Arches
- 6.1- Scope

## 4. Summary and concluding remarks

This paper provides a conceptual overview of recommended design procedures developed from a comprehensive research effort sponsored by the FHWA to arrive at state-of-the-art provisions for the design of noncomposite longitudinally stiffened steel box-section members in the context of AASHTO LRFD. The proposed procedures encapsulate a significant advancement in the understanding of the behavior of longitudinally stiffened box-section members.

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## References

- 1) AASHTO (2017). "AASHTO LRFD Bridge Design Specifications." Eighth Edition, American Association of State Highway and Transportation Officials, Washington DC.
- 2) AISI (2016). "North American Specification for the Design of Cold-Formed Steel Structural Members." American Iron and Steel Institute, Washington, DC.
- 3) AISI (2012). "North American Specification for the Design of Cold-Formed Steel Structural Members." American Iron and Steel Institute, Washington, DC.
- 4) CEN. (2006). "EN 1993-1-5. Eurocode 3—design of steel structures-Part 1-5: Plated structural elements."
- 5) Chou, C. (2011). "Akashi Kaikyo Bridge." Retrieved from <https://www.flickr.com/photos/chitaka/10109544695/>
- 6) GSG (2013). "Lupu Bridge." Retrieved from <https://www.globalsalesgrowth.com>
- 7) Hendy, C.R. and Murphy, C.M. (2007). "Designers' Guide to EN 1993-2 Eurocode 3: Design of Steel Structures, Part 2: Steel Bridges," Thomas Telford, London, 332 pp.
- 8) King C.M. (2017). "A New Design Method for Longitudinally Stiffened Plates." Proceedings of the Annual Stability Conference, Structural Stability Research Council, San Antonio, TX, March.
- 9) King, C. and Brown, D. (2001). "Design of Curved Steel," Steel Construction Institute, Berkshire, 112 pp.
- 10) Lokhande, A. and White, D.W. (2017). "Behavior and Design of Noncomposite Nonlongitudinally Stiffened Welded Steel Box-Section Members." Proceedings of the Annual Stability Conference, Structural Stability Research Council, San Antonio, TX, March.
- 11) Lokhande, A., White, D.W., King, C.M. and Grubb, M.A. (2018). "Improved Characterization of the Flexural and Axial Compressive Resistance of Noncomposite Longitudinally Stiffened Rectangular Welded Steel Box-Section Members." Proceedings of the Annual Stability Conference, Structural Stability Research Council, Baltimore, Maryland, April.
- 12) Lokhande, A. (2018). "Improved Characterization of the Flexural and Axial Compressive Resistance of Welded Steel Box-Section Members." Ph.D. Dissertation, Georgia Institute of Technology (to appear).
- 13) Pekoz, T.B (1986). "Development of a Unified Approach to the Design of Cold-Formed Steel Members." Report SG-86-4, American Iron and Steel Institute, 1986.
- 14) Subramanian, L.P., and D.W. White. (2017). "Improved Strength Reduction Factors for Steel Girders with Longitudinally Stiffened Webs," Research Report, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, January.
- 15) White, D.W., A. Lokhande, C.M. King, M.A. Grubb, A. Ream., and F. Russo (2018). "LRFD Specifications for Noncomposite Steel Box Sections," Report to Federal Highway Administration, FHWA IDIQ DTFH610-14-D-00049 (to appear).
- 16) Ziemian, R.D. (2010). "Guide to Stability Design Criteria for Metal Structures." 6th Edition. New York: John Wiley & Sons, Inc.