A Technical Note: A Direct Method for Obtaining the Plate Buckling Coefficient for Double-Coped Beams

L.S. MUIR and W.A. THORNTON

The AISC LRFD 3rd Edition Manual of Steel Construction (AISC, 2001), hereafter referred to as the AISC Manual, presents a procedure for deep and unequal depth top and bottom coped beams which extends the procedure of the AISC LRFD 2nd Edition Manual of Steel Construction (AISC, 1994), which was limited to shallow \((d_c \leq 0.2d)\), approximately equal depth top and bottom cope. This new procedure involves an interpolation of values from a table to determine the plate buckling coefficient. It is the purpose of this note to present a formula for the plate buckling coefficient which eliminates this interpolation.

FORMULATION

The AISC Manual (AISC, 2001) gives a procedure on page 9-9 for buckling of deep \((d_c > 0.2d)\) and unequal depth \((d_{c_{top}} \neq d_{c_{btm}})\) cope (see the list of symbols in the appendix) as follows.

The design buckling stress \(\phi F_{cr}\) is

\[\phi F_{cr} = 0.9 F_y Q\]

where

\[Q = 1 \text{ for } \lambda \leq 0.7\]
\[= 1.34 - 0.486 \lambda \text{ for } 0.7 < \lambda < 1.41\]
\[= 1.30/\lambda^2 \text{ for } \lambda > 1.41\]

and

\[\lambda = \frac{1}{167} \sqrt{F_y \left( \frac{h_o}{k 2t_w} \right)}\]

(1)

In Equation 1, \(k\), the plate buckling coefficient, must be found by interpolation in Table 1, using the column labeled AISC. These values were scaled from a curve produced by Gerard and Becker (1957), who also gave a formula for \(k\) as,

\[k = \frac{6}{\pi^2} \left[ (1-\nu) + \left( \frac{\pi b m / a}{6} \right)^2 \right]\]

(2)

where

\[b = h_o / 2\]
\[a = c\]
\[m = 1.0 \text{ for a plate simply supported along both loaded edges with one unloaded edge and free along the other unloaded edge}\]

The values for \(k\) from Equation 2 are also shown in Table 1, which shows good agreement between the scaled values and the computed values. Equation 2 can therefore be used to replace the interpolation required in the current AISC procedure.

A further simplification of the AISC Manual procedure can be achieved by substituting Equation 2 into Equation 1, which gives

\[\lambda = \frac{h_o \sqrt{F_y}}{t_w \sqrt{47,500 + 112,000 \left( \frac{h_o}{2c} \right)^2}}\]

(3)

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</table>

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APPENDIX

Symbols

\( a \) = length of plate parallel to the compressive force, denoted as \( c \)

\( b \) = width of plate perpendicular to the compressive force, denoted as \( h_o/2 \)

\( d_c \) = depth of cope

\( h_o \) = reduced beam depth

\( d_{c \text{ top}} \) = depth of top cope

\( d_{c \text{ btm}} \) = depth of bottom cope

\( k \) = plate buckling coefficient, dependent on aspect ratio and boundary conditions of plate

\( m \) = number of half sine waves in buckled plate at minimum compressive stress (\( m=1 \) for a plate simply supported along both loaded edges with one unloaded edge and free along the other unloaded edge)

\( t_w \) = thickness of plate

\( E \) = modulus of elasticity (29,000 ksi for steel)

\( F_{cr} \) = critical buckling stress

\( F_y \) = yield stress

\( \nu \) = Poisson’s ratio (0.3 for steel)

REFERENCES

