91  Investigation on the Performance of a Mathematical Model to Analyze Concentrically Braced Frame Beams with V-Type Bracing Configurations
Alireza Asgari Hadad and Patrick J. Fortney

109  The Indirect Analysis Method of Design for Stability: An Amplifier to Address Member Inelasticity, Member Imperfections, and Uncertainty in Member Stiffness
Rafael Sabelli

Research Update
135  Self-Centering Column Base Connections with Friction Dampers
Judy Liu
Engineering Journal

American Institute of Steel Construction

Dedicated to the development and improvement of steel construction, through the interchange of ideas, experiences and data.

Editorial Staff
Editor Margaret A. Matthew, PE
Managing Editor Keith A. Grubb, SE, PE
Research Editor Judy Liu, PhD
Production Editor Erika Salisbury

Officers
Jack Klimp
Chairman
Stephen Knitter
Vice Chairman
Edward Seglias
Secretary/Legal Counsel
Charles J. Carter, SE, PE, PhD
President
Scott L. Melnick
Senior Vice President
Lawrence F. Kruth, PE
Vice President
Tabitha S. Stine, SE, PE
Vice President
Mark W. Trimble, PE
Vice President

The articles contained herein are not intended to represent official attitudes, recommendations or policies of the Institute. The Institute is not responsible for any statements made or opinions expressed by contributors to this Journal.

The opinions of the authors herein do not represent an official position of the Institute, and in every case the officially adopted publications of the Institute will control and supersede any suggestions or modifications contained in any articles herein.

The information presented herein is based on recognized engineering principles and is for general information only. While it is believed to be accurate, this information should not be applied to any specific application without competent professional examination and verification by a licensed professional engineer. Anyone making use of this information assumes all liability arising from such use.

Manuscripts are welcomed, but publication cannot be guaranteed. All manuscripts should be submitted in duplicate. Authors do not receive a remuneration. Guidelines for authors are printed on the inside back cover.

Engineering Journal (ISSN 0013-8029) is published quarterly. Subscriptions: Members: one subscription, $40 per year, included in dues; Additional Member Subscriptions: $40 per year. Non-Members U.S.: $160 per year. Foreign (Canada and Mexico): Members $80 per year. Non-Members $160 per year. Published by the American Institute of Steel Construction at 130 E Randolph Street, Suite 2000, Chicago, IL 60601.

Periodicals postage paid at Chicago, IL and additional mailing offices.
Postmaster: Send address changes to Engineering Journal in care of the American Institute of Steel Construction, 130 E Randolph Street, Suite 2000, Chicago, IL 60601.

Copyright 2020 by the American Institute of Steel Construction. All rights reserved. No part of this publication may be reproduced without written permission. The AISC logo is a registered trademark of AISC.

Subscriptions: subscriptions@aisc.org, 312.670.2400

Archives: Search at aisc.org/ej. Article downloads are free for current members and are available for a nominal fee for non-members.
Investigation on the Performance of a Mathematical Model to Analyze Concentrically Braced Frame Beams with V-Type Bracing Configurations

ALIREZA ASGARI HADAD and PATRICK J. FORTNEY

In memory of Patrick J. Fortney, who passed away in October 2019.

ABSTRACT

The chevron effect and the corresponding mathematical model used to predict the beam shear force and bending moment demands on beams in concentrically braced frames with V- or inverted V-type (also known as chevrons) configurations is studied in this paper. The common analysis approach considers beam span, work point location, and a concentrated force representing the unbalanced vertical forces of the braces while ignoring any local effects resulting from the brace connection geometry. The assumptions and load distribution mechanisms in the connection region that have been discussed in earlier literature are presented, and the performance of the mathematical model based on chevron effect (CE method) is examined by comparing its results with the results of a net vertical force method (NVF) in addition to those obtained by finite element analysis. The analysis procedure recently proposed for beams in chevron concentrically braced frames is used to design the beams in a group of 20 beam-gusset assemblies. The results revealed the presence of the chevron effect in chevron beams. It also showed the CE method is able to estimate the beam maximum shear force and bending moment. Additionally, the stress distribution along the gusset-to-beam interface is studied.

Keywords: beam, shear force, bending moment, concentrically braced frame, chevron, connection, stress distribution.

INTRODUCTION

Braced frames (concentric and eccentric), moment resisting frames, special truss moment frames, and steel plate shear walls are commonly used as lateral-force-resisting systems in steel structures. While new systems such as buckling restrained braced frames are gaining popularity, concentrically braced frames (CBFs) and moment resisting frames (MRFs) are considered as two of the most popular systems among these alternatives (Azad et al., 2017). CBFs have been quite popular since the 1960s mainly because of their economic advantages over the MRFs, particularly in cases where the drift requirements govern the design and higher stiffness and strength are required (Khatib et al., 1988; Nascimbene et al., 2012; Azad et al., 2017). Studies on the seismic response of braced frames progressed slowly until the 1970s. The expansion of offshore oil exploration in regions of seismic risk stimulated analytical and experimental investigations of the inelastic cyclic response of tubular steel-braced structures. At the same time, evidence was accumulating that buildings with braced frames were being damaged during major earthquakes. Such events motivated an increasing number of studies on the inelastic behavior of steel braced frames both analytically and experimentally (Khatib et al., 1988; Bertero et al., 1989; Lee and Lu, 1989; Wallace and Krawinkler, 1989; Tremblay, 2002; Sabelli et al., 2003; Tremblay et al., 2003; Tremblay, 2008). Special concentrically braced frames (SCBFs) were developed after 1988; they are employed in high seismic regions and use a response modification factor, $R$, to reduce the seismic design force. During large, infrequent earthquakes, SCBFs must sustain cyclic, inelastic tensile yielding and compressive buckling deformation of the brace without significant deterioration of stiffness and resistance for earthquakes exceeding the reduced design force (Sen et al., 2016). Brace buckling and tensile yield are the primary yield mechanisms of the system, which permit the frame to sustain the inelastic deformation and energy dissipation demands needed to provide collapse-prevention performance (Osteraas and Krawinkler, 1989; Naeim, 1998; Roeder et al., 2011; Sen et al., 2016).

For CBFs with buckling braces, the braces are used in opposing pairs to balance the difference in the tensile and compressive capacities. This difference is pronounced in chevron-configured CBFs. In addition to the chevron configuration for CBFs, also known as V- or inverted V-type concentrically braced frames, X-CBFs with braces spanning...
two consecutive floors meeting at one point in the middle beam were also used in CBFs. An inventory of older CBF buildings shows that the chevron brace configuration was common (Zhang et al., 2011; Sloat, 2014; Sen et al., 2016). This is logical because chevron bracing accommodates a wide range of architectural elements, including doors and windows, and fewer brace connections with respect to X-CBFs (Sen et al., 2016; Costanzo et al., 2017). Chevron-braced bays were found to fail in one of two mechanisms: (1) A weak beam mechanism where the buckling of the compression brace causes an unbalanced force to be applied on the beam and subsequently the formation of plastic hinges in the beam. Weak beams lead to considerable plastification in columns, beams, and braces, irrespective of brace slenderness. Slender braces increase the ductility demand in all remaining members, and rarely yield in tension. (2) A strong beam mechanism where buckling of the compression brace is followed by yielding of the tension brace, the beam being sufficiently strong to remain elastic despite the vertical unbalanced force applied to it by the braces at mid-span. Chevron-braced frames with strong beams and weak braces exhibit a more uniform distribution of yielding with height than frames with weak beams and strong braces (D’Aniello et al., 2015). It is not evident from the data in the literature which collapse mechanism is preferable. The weak beam mechanism concentrates the energy dissipation in the beams that are better energy dissipators than the braces. However, it causes a deteriorating force deformation curve, which leads to more energy dissipation demand. The strong beam mechanism provides a trilinear force deformation characteristic that is believed by many to be an ideal force deformation characteristic. On the other hand, it concentrates the energy dissipation in the braces (the least capable elements) and may require unreasonably stiff and strong beams. In addition, it increases the maximum column compression and hence the danger of column buckling. This is a very undesirable situation in that it might lead to column failure, soft-story formation, or incremental collapse (Roeder and Popov, 1978; Rega and Vestroni, 1984; Shen et al., 2015). The mentioned uncertainties regarding the demands on the beam in chevron CBFs with a V- or X-bracing configuration jeopardize the support of the floor and thus make chevron V- and X-bracing undesirable bracing configurations (ASCE, 1998).

In addition to the system behavior in CBFs, brace connection analysis and design and calculation of the connection interface forces have been a subject for several numerical and experimental studies (Thornton, 1984; Astanene-Asl et al., 1985; Bjorhovde and Chakrabarti, 1985; Richard, 1986; Willibald et al., 2006; Shaw et al., 2010; Wigle and Fahnestock, 2010). The uniform force method (UFM) is a popular method typically used to distribute brace forces through the corner connections (Thornton, 1991). However, this method is limited to connections where braces are framed to a beam-column joint. There is very little published work on how to distribute forces in other types of brace configuration. Fortney and Thornton (2015; 2017) studied CBFs with V- and inverted V-type brace configurations. They discussed the behavior of frame beams through a phenomenon called the chevron effect and developed a mathematical method to, initially, determine an admissible distribution of forces acting on the beam-to-gusset interface and, secondly, determine the beam shear force and bending moment in the connection region. It is worth noting that the chevron effect model does not consider any stiffening effect of the gusset plate for the beam. This assumption relies on the results of several experimental tests revealing the out-of-plane bending of the gusset plate when the compression brace buckles (Zhang et al., 2011; Lumpkin et al., 2012; Okazaki et al., 2012; Hsiao et al., 2013; Salawdeh et al., 2017). The mentioned mathematical model will be discussed, and its performance will be studied and compared with finite element model results in the following sections.

**MATHEMATICAL MODEL FOR CHEVRON EFFECT**

The chevron effect is a phenomenon that affects the demands on the frame beam imparted by the distribution of forces at the beam/gusset plate interface. The influence of these interface forces on the beam demands was initially introduced by Fortney and Thornton (2015). The chevron effect is present in CBFs with V- and inverted V-type brace configurations where braces are connected to the frame beam with a gusset plate. The net vertical force (NVF) method is the common method used to analyze the frame beam in such braced frames. However, this method only considers the center line of the columns, beams, and braces and applies the summation of the vertical components of the brace forces to the work point (the point in which the center lines of the braces meet the centerline of the beam) as concentrated load. Fortney and Thornton (2015) developed a mathematical model assuming a uniform distribution of connection interface forces and moments to calculate the shear force and bending moment on the beam. They showed that the presence of a gusset plate influences the force distribution in the connection region and referred to this phenomenon as the chevron effect. Fortney and Thornton (2017) showed the chevron effect results in localized beam shear and moment demands on the frame beam that are not captured by the NVF. Figure 1 presents the chevron effect and the difference between the results of the mathematical model (CE method) and the NVF method for the case in which the tension and compression braces have equal forces (i.e., zero NVF).

The CE method proposed by Fortney and Thornton (2015;
(2017) to analyze the beam in a chevron brace frame does not consider the gusset plate as a stiffening element for the beam. So the flexural deformation of the beam is not influenced by the gusset plates in the connection region. This assumption is based on the instability of the gusset plate at the free edges and the out-of-plane deformation of the gusset plate during brace buckling. The out-of-plane behavior of the gusset plate is observed in several studies. Zhang et al. (2011) studied nine inverted V-braces (W-shapes) and their gusset plate connections under inelastic cyclic loading to define the hysteretic behavior of the assemblies. As can be seen in Figure 2, even the presence of stiffener plates does not impede the out-of-plane response of the gusset plate [see Figure 2(b)].

While the cyclic and monotonic response of isolated components such as braces and gusset plates has been the subject of several studies, the seismic response of two three-story SCBFs was studied to investigate the performance of their structural components when the components work interactively (Lumpkin et al., 2012). The gusset plate rotation and out of plane buckling phenomenon was also observed in this study (see Figure 3). Similar observations are reported by other researchers (Okazaki et al., 2012; Hsiao et al., 2013; Salawdeh et al., 2017).

![Fig. 1. The chevron effect—beam shear and moment (Fortney and Thornton, 2017).](image)

(a) Without stiffener          (b) With stiffener

![Fig. 2. Deformation of gusset plates (Zhang et al., 2011).](image)
**TEST MATRIX**

To evaluate the performance of the CE method, 20 gusset-beam assemblies are designed. Each assembly consists of a beam and either one or two gussets representing the two types of studied CBFs: a chevron (Vb) or X-bracing (Xb) configuration, respectively. The gusset is connected to the beam with two-sided fillet welds.

In order to provide enough freedom for the braces to buckle in real practice, a clearance must be provided between the end of the brace-to-gusset weld and the beam-to-gusset interface. Two values of clearances (2.0 in. and 8.0 in.) are considered in the models in order to evaluate the effect of the clearance distance on the stress distribution of the welded connection interface. The 2.0-in. clearance is assumed to represent a clearance that might be used when the braces frame to the bottom side of a beam. The 8.0-in. clearance is assumed to represent a clearance that might be used when a gusset frames to the top side of a beam (e.g., if one wanted the end of the brace to clear a floor slab). Models identified with an uppercase L, such as “M1 L”, are the models with the 8.0-in. clearance.

Three types of loads are considered in the design of the gussets and the beam-to-gusset welds: (1) equal tension and compression brace forces (Equation 1); (2) mechanism load (Mec), which is the expected tensile strength of the tension brace and the expected buckling strength of the compression brace (in the case of X-bracing configuration, it will be the yield of two braces on a same line and buckling of the other two braces on the other line); and (3) the expected post-buckling strength of the compression brace (Pst) and the expected tensile strength of the tension brace. The brace forces are calculated using Equations 1–3 as required in AISC Seismic Provisions Section F2.6c (AISC, 2016).

\[ P_{\text{tension}} = R_{u}F_{y}A_{g} \]  
\[ P_{\text{compression}} = 1.14F_{\text{cre}}A_{g} \]  
\[ P_{\text{post-buckling}} = 0.3P_{\text{compression}} \]

For sizing the beam, the maximum shear force and bending moment are obtained based on the CE method. The most economic (lightest) section is selected from Part 1 of the AISC Steel Construction Manual (AISC, 2017). Figure 4 shows the process of designing the beam using the CE method and compares it with the NVF method. The main difference between the two methods is that the NVF method is not dependent on the beam size and gusset geometry whereas the CE method is. Therefore, the CE method is an iterative process whereas the NVF method is not. The governing limit state for each beam used in each assembly is provided in Table 1.

Gusset thickness is determined based on eight limit states, including:
1. Tension along the brace line based on Whitmore section.
2. Compression along the brace line based on the Whitmore section.
3. Normal (axial) force to section a-a.
4. Shear force along section a-a.
5. Bending moment on section a-a.
6. Normal (axial) force to section b-b.
7. Shear force along section b-b.
8. Bending moment on section b-b.

Figure 5 shows the design parameters.

![Fig. 3. Mid-span gusset rotation due to brace buckling (Lumpkin et al., 2012).](image-url)
Fig. 4. Comparison of chevron beam design methods.

Fig. 5. Design parameters (Fortney and Thornton, 2015).
**FINITE ELEMENT MODELS:**

Abaqus 6.13 (Simulia, 2013) is employed to model and analyze the gusset-beam assemblies (listed in Table 1) and evaluate beam shear force and bending moment. An eight-node linear brick element is used for all parts in the models. An elastic perfectly plastic material model is used for all parts: ASTM A992 for the beams \((F_y = 50 \text{ ksi})\), ASTM A529 Gr. 50 for the gussets \((F_y = 50 \text{ ksi})\), and E70 \((F_u = 70 \text{ ksi})\) for the welds. Two-sided fillet welds are used to connect the gusset(s) to the beam. Different mesh sizes were studied for each of the parts to find the appropriate mesh size. The mesh size study is summarized in Figure 6. The selected mesh sizes are 1 in. for the beam, 0.3 in. for the gusset, and 0.2 in. for the weld materials.

### Table 1. List of FEA Models with Design Details

<table>
<thead>
<tr>
<th>No.</th>
<th>Model</th>
<th>Clearance (in.)</th>
<th>Load</th>
<th>Beam</th>
<th>Beam Limit State</th>
<th>Gusset Limit State</th>
<th>(t_{gusset}) (in.)</th>
<th>(D) (in.)</th>
<th>Geometry (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>M1</td>
<td>2</td>
<td>Eql</td>
<td>W14×26</td>
<td>V</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>2</td>
<td>M1 L</td>
<td>8</td>
<td>Eql</td>
<td>W12×26</td>
<td>V</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>3</td>
<td>M2</td>
<td>2</td>
<td>Eql</td>
<td>W18×65</td>
<td>V</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>4</td>
<td>M2 L</td>
<td>8</td>
<td>Eql</td>
<td>W16×50</td>
<td>V</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>5</td>
<td>M3</td>
<td>2</td>
<td>Mec</td>
<td>W30×108</td>
<td>M</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>6</td>
<td>M3 L</td>
<td>8</td>
<td>Mec</td>
<td>W30×108</td>
<td>M</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>7</td>
<td>M3 Pst</td>
<td>2</td>
<td>Pst</td>
<td>W30×108</td>
<td>M</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>8</td>
<td>M3 L Pst</td>
<td>8</td>
<td>Pst</td>
<td>W30×108</td>
<td>M</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>9</td>
<td>M4</td>
<td>2</td>
<td>Mec</td>
<td>W24×84</td>
<td>V</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>10</td>
<td>M4 L</td>
<td>8</td>
<td>Mec</td>
<td>W21×62</td>
<td>V</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>11</td>
<td>M4 Pst</td>
<td>2</td>
<td>Pst</td>
<td>W24×84</td>
<td>V</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>12</td>
<td>M4 L Pst</td>
<td>8</td>
<td>Pst</td>
<td>W21×62</td>
<td>V</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>13</td>
<td>M5</td>
<td>2</td>
<td>Mec</td>
<td>W24×76</td>
<td>V</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>14</td>
<td>M5 L</td>
<td>8</td>
<td>Mec</td>
<td>W21×55</td>
<td>V</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>15</td>
<td>M5 Pst</td>
<td>2</td>
<td>Pst</td>
<td>W24×76</td>
<td>V</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>16</td>
<td>M5 L Pst</td>
<td>8</td>
<td>Pst</td>
<td>W21×55</td>
<td>V</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>17</td>
<td>M6</td>
<td>2</td>
<td>Mec</td>
<td>W24×84</td>
<td>V</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>18</td>
<td>M6 L</td>
<td>8</td>
<td>Mec</td>
<td>W21×62</td>
<td>V</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>19</td>
<td>M6 Pst</td>
<td>2</td>
<td>Pst</td>
<td>W24×84</td>
<td>V</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>20</td>
<td>M6 L Pst</td>
<td>8</td>
<td>Pst</td>
<td>W21×62</td>
<td>V</td>
<td>C</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
</tbody>
</table>

**Notes:**
- Eql: Equal tension and compression loading
- Mec: Mechanism (buckling) loading
- Pst: Post-buckling loading
- \(A_{b-b}\): Axial load on gusset section b-b
- C: Compression
- T: Tension
- R: Rectangular
- W: Whitmore
- H: Story height
- V: Shear
- M: Moment (bending)
- Vb: Chevron bracing configuration
- Xb: X bracing configuration
- \(a\): Distance from work-point to left beam end
- \(b\): Distance from work-point to right beam end
- \(\Delta\): Horizontal eccentricity
- \(t_{gusset}\): Gusset thickness
ANALYSIS RESULTS

The shear force and bending moment diagrams of the beams were extracted from the finite element analysis (FEA) and plotted to compare the prediction of the CE and NVF methods to that of the FEA. Because the maximum beam shear force and bending moment are the important values in designing the beam section, a summary of the maximum values is reported in Table 2. To discuss and compare the results in more detail, all models are categorized into four groups based on the summation of the braces' vertical forces and the number of gusset plates used in each model:

1. Group 1: \( \Sigma V_T = 0 \); gusset plate on one side of the beam
2. Group 2: \( \Sigma V_T = 0 \); gusset plate on both sides of beam
3. Group 3: \( \Sigma V_T \neq 0 \); gusset plate on one side of beam
4. Group 4: \( \Sigma V_T \neq 0 \); gusset plates on both sides of beam

The results of the analyses for each group are presented in the following sections.

**Group 1: Summation of Vertical Forces = Zero; Single Gusset Plate**

Because the summation of vertical forces is zero in this group, the NVF method does not predict any shear force and bending moment on the beam. However, due to the presence of the gusset, there is a shear force and bending moment on the beam due to the chevron effect, which is also observed in the FEA results. As can be seen in Table 3, the CE method predicts the maximum beam shear force and bending moment with values close to the FEA values. On average, the maximum beam shear force derived by CE method is 20% (1.22) larger than that of the FEA, and the maximum beam moment is approximately equal the FEA value (0.96). The distribution of beam shear force and bending moment for all methods is plotted in Figure 7 for model M1.

![Fig. 6. Mesh size determination analysis for different parts.](image-url)
Group 2: Summation of Vertical Forces = Zero; Double Gusset Plates

Similar to the first group’s models, the summation of vertical forces is zero in this group, and thus the NVF does not predict any beam shear force and bending moment. As presented in Table 4, the CE method predicts a beam maximum shear force approximately 20% larger than that of the FEA. There is a big difference, however, between the maximum bending moment predicted by CE method versus the one in the FEA; the CE maximum bending moment is, on average, 5.5 times the FEA value. The difference is due to the presence of double gusset plates and the unavoidable suggestion of the FEA to stiffen the beam section with the gusset plates. It influences the beam flexural deformation, which leads to different beam bending moment values. The mentioned view point on the gusset plate(s) opposes the CE method that assumes (based on the discussed experimental observations) the gusset plate(s) does not stiffen up the beam section and thus does not have any impact on the magnitude of the beam bending moment. Regardless of the mentioned issue, the CE method can yet be a useful tool in the analysis and designing of such beams because the shear force is the governing limit state in selecting the beam section and the NVF method is not able to predict any shear force demand on the beam. The analysis results of model M2 is plotted in Figure 8 as an example.
Fig. 7. Model M1 results.
Table 4. Summation of Vertical Forces = Zero; Double Gusset Plates

<table>
<thead>
<tr>
<th>No.</th>
<th>Model</th>
<th>CE/FEA</th>
<th>NVF/FEA</th>
<th>NVF/CE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$V_{\text{max}}$ (kips)</td>
<td>$M_{\text{max}}$ (kip-in.)</td>
<td>$V_{\text{max}}$ (kips)</td>
</tr>
<tr>
<td>3</td>
<td>M2</td>
<td>1.24</td>
<td>2.85</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>M2 L</td>
<td>1.31</td>
<td>6.51</td>
<td>0.00</td>
</tr>
<tr>
<td>9</td>
<td>M4</td>
<td>1.23</td>
<td>3.10</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td>M4 L</td>
<td>1.26</td>
<td>9.02</td>
<td>0.00</td>
</tr>
<tr>
<td>11</td>
<td>M4 Pst</td>
<td>1.24</td>
<td>3.07</td>
<td>0.00</td>
</tr>
<tr>
<td>12</td>
<td>M4 L Pst</td>
<td>1.27</td>
<td>8.64</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>1.26</td>
<td>5.53</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Fig. 8. Model M2 results.
Group 3: Summation of Vertical Forces ≠ Zero; Single Gusset Plate

Based on the results in Table 5, the NVF method can predict the beam maximum bending moment in the models in which the summation of the vertical forces is not zero, but this method does not predict the beam maximum shear force. On the other hand, the CE method estimates a beam maximum shear force; approximately corresponding to the FEA (1.09) and bending moment 20% larger than FEA results. Thus, the CE method can be considered as a more versatile tool in analyzing chevron beams. The results of model M3 in this group is plotted in Figure 9.

<table>
<thead>
<tr>
<th>No.</th>
<th>Model</th>
<th>CE/FEA</th>
<th>NVF/FEA</th>
<th>NVF/CE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>v_{max} (kips)</td>
<td>M_{max} (kip-in.)</td>
<td>v_{max} (kips)</td>
<td>M_{max} (kip-in.)</td>
</tr>
<tr>
<td>5</td>
<td>M3</td>
<td>1.16</td>
<td>1.21</td>
<td>0.387</td>
</tr>
<tr>
<td>6</td>
<td>M3 L</td>
<td>1.13</td>
<td>1.25</td>
<td>0.447</td>
</tr>
<tr>
<td>7</td>
<td>M3 Pst</td>
<td>1.07</td>
<td>1.16</td>
<td>0.803</td>
</tr>
<tr>
<td>8</td>
<td>M3 L Pst</td>
<td>0.992</td>
<td>1.19</td>
<td>0.875</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>1.09</td>
<td>1.20</td>
<td>0.628</td>
</tr>
</tbody>
</table>

Group 4: Summation of Vertical Forces ≠ Zero; Double Gusset Plates

Due to the presence of double gusset plates and the discussed issue regarding the stiffening influence of the gusset plates on the beam section, the CE method predicts the maximum beam moment on average 80\% greater than that of the FEA. Because the beam maximum shear force is the determinative limit state in designing the X-bracing chevron beams (double gusset plate), and the CE method can estimate the maximum beam shear force with 20\% difference in comparison to the FEA; the CE method can be used in analyzing such beams. Note that the NVF method does not

---

**Table 5. Summation of Vertical Forces ≠ Zero; Single Gusset Plate**

<table>
<thead>
<tr>
<th>No.</th>
<th>Model</th>
<th>CE/FEA</th>
<th>NVF/FEA</th>
<th>NVF/CE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>v_{max} (kips)</td>
<td>M_{max} (kip-in.)</td>
<td>v_{max} (kips)</td>
<td>M_{max} (kip-in.)</td>
</tr>
<tr>
<td>5</td>
<td>M3</td>
<td>1.16</td>
<td>1.21</td>
<td>0.387</td>
</tr>
<tr>
<td>6</td>
<td>M3 L</td>
<td>1.13</td>
<td>1.25</td>
<td>0.447</td>
</tr>
<tr>
<td>7</td>
<td>M3 Pst</td>
<td>1.07</td>
<td>1.16</td>
<td>0.803</td>
</tr>
<tr>
<td>8</td>
<td>M3 L Pst</td>
<td>0.992</td>
<td>1.19</td>
<td>0.875</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>1.09</td>
<td>1.20</td>
<td>0.628</td>
</tr>
</tbody>
</table>

---

**Fig. 9. Model M3 results.**
estimate the maximum beam shear force in such models (see the maximum shear ratios in Table 6). Model M5 results are plotted in Figure 10.

**PERFORMANCE OF THE CE METHOD IN THE CASE OF FLAT BAR GUSSETS**

As discussed earlier, the FEA suggests that the gusset plate(s) stiffen the beam influencing the flexural deformation of the beam and thus the beam bending moment values. In order to remove this influence, model M1 is reconsidered with one difference; the rectangular gusset plate is replaced with a couple of flat bar gusset plates. The new model’s geometry and the analysis results are shown in Figure 11. According the beam shear force and bending moment results, the CE method correlates very well with the FEA when the stiffening influence of the gusset plate on the beam section is removed. As such, the assumption of uniformly distributed

<table>
<thead>
<tr>
<th>No.</th>
<th>Model</th>
<th>CE/FEA</th>
<th>NVF/FEA</th>
<th>NVF/CE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$V_{max}$ (kips)</td>
<td>$M_{max}$ (kip-in.)</td>
<td>$V_{max}$ (kips)</td>
</tr>
<tr>
<td>13</td>
<td>M5</td>
<td>1.25</td>
<td>1.40</td>
<td>0.175</td>
</tr>
<tr>
<td>14</td>
<td>M5 L</td>
<td>1.15</td>
<td>1.35</td>
<td>0.225</td>
</tr>
<tr>
<td>15</td>
<td>M5 Pst</td>
<td>1.25</td>
<td>1.40</td>
<td>0.176</td>
</tr>
<tr>
<td>16</td>
<td>M5 L Pst</td>
<td>1.09</td>
<td>1.34</td>
<td>0.227</td>
</tr>
<tr>
<td>17</td>
<td>M6</td>
<td>1.19</td>
<td>2.30</td>
<td>0.026</td>
</tr>
<tr>
<td>18</td>
<td>M6 L</td>
<td>1.21</td>
<td>2.20</td>
<td>0.037</td>
</tr>
<tr>
<td>19</td>
<td>M6 Pst</td>
<td>1.19</td>
<td>2.31</td>
<td>0.026</td>
</tr>
<tr>
<td>20</td>
<td>M6 L Pst</td>
<td>1.21</td>
<td>2.20</td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td><strong>Average</strong></td>
<td>1.19</td>
<td>1.81</td>
<td>0.116</td>
</tr>
</tbody>
</table>

*Fig. 10. Model M5 results.*
loads acting at the flat bar-beam interfaces is a valid assumption for flat bar gusset connections as well.

UNIFORM VERSUS NONUNIFORM STRESS DISTRIBUTION:
One of the common issues in brace connections is the uneven stress distribution across the gusset edge (Richard, 1986). In the case of a bracing connection welded to the flange of a member, which has significantly more rigidity than a connection to the web, the potential inability of the system to accommodate force distribution was observed (Hewitt and Thornton, 2004). The development of a peak stress induced at some point across the welded connection might cause the weld to fail at the point where the stress is concentrated, causing an unzipping of the weld and a progressive failure of the welded connection (Hewitt and Thornton, 2004). Such a phenomenon has been confirmed in experimental studies as well. See Figure 12 as an example (Lumpkin et al., 2012). To solve this issue, Richard (1986) considered the ratio of peak over average of the stress distribution at the edges of the studied gussets. In response to the Richard (1986) study, AISC introduced a weld ductility factor, equal to 1.4, that was intended to allow for adequate stress redistribution in the weld across the interface. That factor was subsequently reduced to 1.25 (Hewitt and Thornton, 2004).

![Model geometry](image)

![Deformed shape](image)

![Beam shear force diagram](image)

![Beam bending moment diagram](image)

Fig. 11. Model M1 with flat bar gusset plates results.
Stress concentration at the edges of the gusset(s) to the weld interface was observed in all finite element models studied in this research. Figure 13 shows a sample presenting the stress concentration at the left edge of the gusset-to-weld connection. In order to check the stress distribution along the gusset-to-weld connection region, a section cut is considered along the bottom of the gusset plate (above the fillet weld), and the vertical stress distribution at the section cut is studied. Finally, the peak over average ratio of the vertical stress distribution along the gusset to weld connection region is evaluated for all of the models.

Figure 14 shows the variation of peak to average ratio in 67 welded gusset connections. These values are also tabulated in Table 7. In Table 7, the ratios are given for the left and right half (compression and tension zones) of the gussets. The average of all ratios is 2.38, and the standard deviation is 0.63. These results show that the 1.4 ductility factor originally required for the corner gusset plates should be increased to a larger value (approximately 3) in the case of middle gusset plates used in chevron configuration. Another possible approach to solve this problem is to design the welds based on the shear strength of the gusset plate. Further studies with regard to the determination of the ductility factor for the case of chevron gusset plate connections are recommended by the authors.

(a) During the test  
(b) End of the test

Fig. 12. Weld tearing at corner gusset plate (Lumpkin et al., 2012).

Fig. 13. Vertical stress distribution at gusset to weld interface, model M6.
Table 7. Vertical Stress Distribution Ratios

<table>
<thead>
<tr>
<th>No.</th>
<th>Model</th>
<th>Top Gusset</th>
<th>Bottom Gusset</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Peak/Average</td>
<td>Peak/Average</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Left Half</td>
<td>Right Half</td>
</tr>
<tr>
<td>1</td>
<td>M1</td>
<td>1.74</td>
<td>1.89</td>
</tr>
<tr>
<td>2</td>
<td>M1 L</td>
<td>3.87</td>
<td>2.13</td>
</tr>
<tr>
<td>3</td>
<td>M2</td>
<td>1.96</td>
<td>2.01</td>
</tr>
<tr>
<td>4</td>
<td>M2 L</td>
<td>2.18</td>
<td>2.16</td>
</tr>
<tr>
<td>5</td>
<td>M3</td>
<td>2.10</td>
<td>2.61</td>
</tr>
<tr>
<td>6</td>
<td>M3 L</td>
<td>2.54</td>
<td>1.88</td>
</tr>
<tr>
<td>7</td>
<td>M3 Pst</td>
<td>1.99</td>
<td>1.79</td>
</tr>
<tr>
<td>8</td>
<td>M3 L Pst</td>
<td>2.36</td>
<td>1.41</td>
</tr>
<tr>
<td>9</td>
<td>M4</td>
<td>2.24</td>
<td>2.03</td>
</tr>
<tr>
<td>10</td>
<td>M4 L</td>
<td>2.03</td>
<td>2.27</td>
</tr>
<tr>
<td>11</td>
<td>M4 Pst</td>
<td>2.43</td>
<td>3.76</td>
</tr>
<tr>
<td>12</td>
<td>M4 L Pst</td>
<td>2.26</td>
<td>2.70</td>
</tr>
<tr>
<td>13</td>
<td>M5</td>
<td>2.18</td>
<td>2.80</td>
</tr>
<tr>
<td>14</td>
<td>M5 L</td>
<td>2.70</td>
<td>1.90</td>
</tr>
<tr>
<td>15</td>
<td>M5 Pst</td>
<td>2.68</td>
<td>2.26</td>
</tr>
<tr>
<td>16</td>
<td>M5 L Pst</td>
<td>3.22</td>
<td>1.65</td>
</tr>
<tr>
<td>17</td>
<td>M6</td>
<td>2.38</td>
<td>1.94</td>
</tr>
<tr>
<td>18</td>
<td>M6 L</td>
<td>2.58</td>
<td>1.34</td>
</tr>
<tr>
<td>19</td>
<td>M6 Pst</td>
<td>2.44</td>
<td>1.93</td>
</tr>
<tr>
<td>20</td>
<td>M6 L Pst</td>
<td>3.34</td>
<td>—</td>
</tr>
</tbody>
</table>

Fig. 14. Variation of peak over average vertical stress ratio in all assemblies.
CONCLUSION

The chevron effect is a phenomenon that occurs in the frame beams of CBFs with chevron V- and inverted V-bracing configurations due to the presence of gusset plate(s). This phenomenon was introduced in earlier literature, and a mathematical model was developed to generate an admissible force distribution in the chevron gusset connection to obtain beam shear force and bending moment with the inclusion of the chevron effect. The work reported in this paper considered 20 beam-gusset assemblies connected by fillet welds. To check the performance of the mathematical model (CE method), the authors used finite element analysis (FEA) to investigate the beam shear force and bending moment of all models.

The FEA results clearly revealed the presence of the chevron effect in the beam shear force and bending moment diagrams. The net vertical force (NVF) method, which has been commonly used in analyzing chevron beams, was not able to capture the chevron effect and could not estimate the maximum beam shear force accurately in all models, while the CE method could capture the chevron effect. On average, the maximum beam shear force estimated by the CE method is approximately 20% higher than that in the FEA results. Thus, the CE method can always estimate the beam shear force with an acceptable conservatism.

Due to the out-of-plane behavior of the gusset plate, the CE method does not consider the gusset plate(s) as a stiffening element for the beam section. However, the FEA stiffens the beam section by the gusset plate(s), which influences the beam flexural deformation and thus the value of bending moment. The mentioned issue caused the difference between the CE method and the finite element model bending moment results, specifically for models with double gusset plates. On average, the ratio between the CE maximum bending moment and that of the FEA analysis is 2.4. However, because the maximum beam shear force is the governing limit state in cases with X-bracing configurations (double gusset plates), the CE method reasonably predicts the maximum beam shear force in such models and can be a useful tool in designing all types of chevron brace frame beams. If one can reasonably ensure that the gusset will experience no out-of-plane deformations, a rational analysis of the beam that includes the gusset stiffening effects may be reasonable.

The stress concentrations at the ends of the gusset-to-beam welds, caused local failure at the weld terminations. To avoid this local failure, the ratio of peak over average of the vertical stress distribution along the gusset to weld connection interface is calculated and studied. The results showed the ductility factor calculated for corner gussets should be reevaluated in the case of middle gusset plates. Until a more detailed study is performed to evaluate the interface stresses, the authors recommend providing an interface weld that develops the shear strength of the gusset plate.

IN MEMORY OF PATRICK FORTNEY

The impact of losing Dr. Patrick Fortney during the course of this research is beyond words. Kind, knowledgeable, dedicated, and thoughtful are the words that are being echoed by his friends and colleagues when remembering and describing him. His knowledge in structural engineering—next to his experience in steel structure design, construction, and fabrication—made him the pioneering engineer to identify the chevron effect in the behavior of chevron beams in concentrically braced frames. Although his loss has left a great void in the steel industry, as well as his friends’ and colleagues’ hearts, his scientific findings and unique professional contributions will be remembered vividly forever.

SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_g$</td>
<td>Gross cross-section area of beam, in.$^2$</td>
</tr>
<tr>
<td>$A_{b-b}$</td>
<td>Normal (axial) force to section b-b limit state</td>
</tr>
<tr>
<td>$C$</td>
<td>Compression along the brace line limit state</td>
</tr>
<tr>
<td>$E_{ql}$</td>
<td>Equal tension and compression brace loads on the assembly</td>
</tr>
<tr>
<td>$F_{cr}$</td>
<td>Critical stress, ksi</td>
</tr>
<tr>
<td>$F_u$</td>
<td>Specified minimum tensile strength, ksi</td>
</tr>
<tr>
<td>$F_y$</td>
<td>Specified minimum yield stress, ksi</td>
</tr>
<tr>
<td>$H_1$</td>
<td>Horizontal component of force in brace 1, kips</td>
</tr>
<tr>
<td>$H_2$</td>
<td>Horizontal component of force in brace 2, kips</td>
</tr>
<tr>
<td>$L_1$</td>
<td>Horizontal distance from the left edge of the gusset to the work point, in.</td>
</tr>
<tr>
<td>$L_2$</td>
<td>Horizontal distance from the right edge of the gusset to the work point, in.</td>
</tr>
<tr>
<td>$L_g$</td>
<td>Contact length of the gusset to beam interface, in.</td>
</tr>
<tr>
<td>$M$</td>
<td>Beam bending moment limit state</td>
</tr>
<tr>
<td>$M_{a-a}$</td>
<td>Moment acting at the gusset to beam interface, kip-in.</td>
</tr>
<tr>
<td>$M_{max}$</td>
<td>Beam maximum bending moment, kip-in.</td>
</tr>
<tr>
<td>$M_{ec}$</td>
<td>Mechanism (buckling) loading on the assembly</td>
</tr>
<tr>
<td>$P_{compression}$</td>
<td>Brace compression strength, kips</td>
</tr>
<tr>
<td>$P_{post-buckling}$</td>
<td>Post-buckling strength, kips</td>
</tr>
</tbody>
</table>
P_tension \quad \text{Brace tensile strength, kips}

Pst \quad \text{Post-buckling loading on the assembly}

T \quad \text{Tension along the brace line limit state}

R_y \quad \text{Ratio of the expected yield stress to the specified minimum yield stress}

V \quad \text{Beam shear force limit state}

V_1 \quad \text{Vertical component of force in brace 1, kips}

V_2 \quad \text{Vertical component of force in brace 2, kips}

V_{max} \quad \text{Maximum beam shear force, kips}

Vb \quad \text{V-bracing configuration}

Xb \quad \text{X-bracing configuration}

a \quad \text{Distance from left beam support to location of work point, in.}

b \quad \text{Distance from work point to right beam support, in.}

e_b \quad \text{Perpendicular distance from the gusset interface to the gravity axis of the frame beam, in.}

h \quad \text{Vertical distance of the gusset, in.}

t_{gusset} \quad \text{Gusset plate thickness}

w_1 \quad \text{Net uniformly distributed transverse load on left half of gusset, kip/in.}

w_2 \quad \text{Net uniformly distributed transverse load on right half of gusset, kip/in.}

w.p. \quad \text{Brace work point}

Δ \quad \text{Horizontal misalignment between the work point and the centroid of the gusset-to-beam interface, in.}

ΣV_T \quad \text{Summation of vertical components of brace forces, kips}

REFERENCES


The Indirect Analysis Method of Design for Stability: An Amplifier to Address Member Inelasticity, Member Imperfections, and Uncertainty in Member Stiffness

RAFAEL SABELLI

ABSTRACT

Design for stability requires multiple considerations, including geometric second-order effects ($P-\Delta$ and $P-\delta$ effects) and the effects on structure response of member inelasticity, member imperfections, and uncertainty in member stiffness. The “indirect analysis method” provides a simple amplifier approach to addressing these effects such that effective length factors may be taken as 1.0. The indirect analysis method meets the stability design requirements of the AISC Specification and can reduce analysis and design effort for many typical building structures.

Keywords: stability, direct analysis method, effective length, first-order analysis.

The indirect analysis method (IAM) provides a simple amplifier approach to addressing the effects on structure response of member inelasticity (including the effects of residual stress), member imperfections, and uncertainty in member stiffness. [Together the effects of these three factors (inelasticity, member imperfection, and stiffness uncertainty) are referred to here as stiffness-reduction effects.] The IAM permits second-order analysis with full stiffness properties; stiffness-reduction effects are addressed by an amplifier ($B_3$) applied to lateral loads prior to analysis or to their effects afterward. Verification studies confirm design forces consistent with the AISC Specification direct analysis method over a broad range of multiple parameters. The indirect analysis method can also be used as the basis of an improved first-order analysis method.

CURRENT METHODS OF DESIGN FOR STABILITY IN THE AISC SPECIFICATION

The AISC Specification for Structural Steel Buildings, hereafter referred to as the AISC Specification, defines three methods of design for stability (AISC, 2016): the direct analysis method (the DM, presented in Chapter C), the effective-length method (the ELM, presented in Appendix 7, Section 7.2), and the first-order analysis method (the FOM, presented in Appendix 7, Section 7.3). Each method includes coordinated analysis and design requirements that result in structures with sufficient strength to prevent instability, considering the structures’ stiffness with respect to lateral displacement in the presence of vertical loads. Each method addresses stiffness-reduction effects differently, and each method requires either a second-order analysis (DM and ELM) or determination of the second-order effect (FOM). Second-order analysis may be either explicit or approximate, such as by means of the $B_1$ and $B_2$ factors [where $B_1$ and $B_2$ are the second-order effect multipliers for $P-\Delta$ and $P-\delta$ effects, respectively, defined in AISC Specification Appendix 8 (AISC, 2016)].

The DM addresses stiffness-reduction effects in the analysis, requiring a second-order analysis using reduced member stiffness. This produces the internal member forces that are expected to result from the application of the external forces, considering stiffness-reduction effects on the response of the structure. Stiffness-reduction effects in the DM are addressed both globally and member by member. Globally, all stiffness terms are reduced using a factor of 0.8. Additionally, the member flexural stiffness reduction for moment frame columns includes a term, $\tau_b$, that relates to the member axial force, and thus at least one analysis iteration is required to determine the axial force corresponding to the loading condition. (Each load combination could entail its own particular set of member stiffness reductions, although grouping is typically done to reduce design effort.) Nevertheless, because the reduced stiffness of the DM model is not intended to be used for calculating drifts, a minimum of two models is typically required for building design: one for strength verification and another for drift determination [and period determination for seismic loads (ASCE, 2016)]. (Drift may be a service limit state, as is often the case for wind, or a safety limit state, as is the...
case for seismic; in either case, full stiffness properties are used, although for different reasons.) With the DM, stability is adequately addressed without the need for an effective length factor, $K$, greater than 1.0. The DM defines notional lateral loads to represent initial system imperfections due to out-of-plumbness. These notional loads are required for gravity-only load combinations; they are also required for combinations that include other lateral loads for which the ratio of second-order load to first-order load exceeds 1.7 in the reduced-stiffness DM analysis.

The ELM addresses stiffness-reduction effects by requiring reduced compression strength (implemented by effective length factors in the design of columns) and addresses $P-\Delta$ effects by requiring a second-order analysis. The flexural demands imposed by columns that are near their stability limit on the column connections and on beams are not directly addressed in the ELM. This method allows for effective length factors to be taken as 1.0 for braced-frame columns and for moment-frame columns when $B_2 \leq 1.1$; determination of effective length factors for moment-frame columns is required when $B_2 > 1.1$. Several methods exist to account for the effect of leaning columns in determining effective length factors, with minor variation in the results (Geschwindner, 2002). The ELM is limited to cases in which the second-order effect does not exceed 1.5; gravity-only load combinations require notional loads. (The limit of 1.5 for a full-stiffness ELM model is equivalent to 1.7 for a reduced-stiffness DM analysis.)

In the FOM, stiffness-reduction and $P-\Delta$ effects are addressed together in the analysis by means of an additional lateral force in a first-order analysis. This force is derived in AISC Design Guide 28, Appendix B: “Development of the First-Order Analysis Method” (Griffis and White, 2013). To permit this simplification, limits are placed on both axial force in moment-frame columns and on second-order effects. FOM additional forces assume that second-order effects are at the maximum level permitted: $B_2 = 1.5$. Similar to the DM, the FOM allows for effective length factors to be taken as 1.0. Application of the additional lateral forces in the FOM for strength design does not guarantee that second-order effects on drift have been adequately addressed when consideration of these is required, such as by ASCE/SEI 7, Section 12.8.7 (ASCE, 2016), unless these forces are included in the drift analysis. Inclusion of such forces in the drift analysis is problematic for two reasons: Drift is typically checked in a separate analysis not subject to the same minima as the analysis for strength design; only geometric second-order effects, not stiffness-reduction effects, are required in drift determination (ASCE, 2016). While the FOM does not require a second-order analysis, its use requires that the second-order effect not exceed 1.5. Confirmation of this requires either a second-order analysis or calculation of the $B_2$ factor, and thus only modest additional effort would be required to incorporate second-order effects into the analysis.

THE INDIRECT ANALYSIS METHOD
The IAM is a simplified version of the DM, with stiffness-reduction effects addressed either globally or story by story rather than member by member. The IAM requires only relatively easy-to-implement means (second-order analysis and an amplification factor applied to lateral loads or to their effects) and does not require more cumbersome methods (such as determination of effective length factors greater than 1.0 or multiple models). The IAM incorporates both second-order analysis (either explicit second-order analysis or first-order analysis amplified by $B_1$ and $B_2$ factors) and an amplification factor to address stiffness reduction ($B_3$). This amplification factor is derived to amplify the second-order analysis forces such that the results obtained match those from the DM. Thus, the $B_1$ factor effectively performs the same function as the 0.8$Lc$ stiffness reduction in the DM, permitting the use of effective length factors, $K$, of 1.0 in the IAM. Use of a global or story value of $\tau_b$ may make the IAM more conservative than the DM for cases with a wide range of moment-frame column axial force ratios.

General Stability Design Requirements
The AISC Specification requires five considerations for any admissible stability design method. These are listed in the following, along with the description of how each is addressed in the IAM. The approach to each consideration closely matches that for the DM, with the difference that items 3–5 utilize the $B_3$ factor, which incorporates 0.8$Lc$, to address stiffness-reduction effects.

1. Consider all deformations. This requirement is addressed by proper modeling.

2. Consider second-order effects (including $P-\Delta$ and $P-\delta$ effects). This requirement is addressed by performing a second-order analysis, either explicit or through application of $B_1$ and $B_2$ factors.

3. Consider geometric imperfections. This requirement is addressed by application of notional loads (for the effect of structural imperfections on the structural response), application of the $B_1$ factor (for the effect of member imperfections on the structural response), and application of member strength formulae with $L_c = L$ (for the effect of member imperfections on member strength).

4. Consider stiffness reduction due to inelasticity. This requirement is addressed by application of the $B_3$ factor (for the effect on the structural response) and application of member strength formulae with $L_c = L$ (for the effect on member strength).
5. **Consider uncertainty in strength and stiffness.** This requirement is addressed by application of the $B_2$ factor (for the effect on the structural response) and application of member available strength formulae with $L_e = L$ (for the effect on member strength).

### General Design Guidance

Although the investigation of stability effects tends to address systems near instability, examination of such cases should not be taken as endorsement of designing systems near instability. The limits proposed for the IAM are based solely on the verification of the method. As systems approach those limits, they become increasingly sensitive to the accuracy of loading assumptions, modeling, and analysis. Many engineers—including the author—apply lower limits on second-order and stiffness-reduction effects to their designs. Furthermore, it should be noted that for seismic design, second-order and stiffness-reduction effects to their designs. Therefore, it should be noted that for seismic design, ASCE/SEI 7, Section 12.8.7, limits the stability coefficient, $\theta$, to 0.25 (or lower, for many systems). According to the ASCE/SEI 7 commentary (ASCE, 2016), the stability coefficient (using symbols from the AISC Specification) can be expressed as:

$$\theta = \frac{\alpha P_{story} \Delta H}{HL}$$

where

- $H$ = total story shear, kips
- $L$ = story height, in.
- $P_{story}$ = total vertical load supported by the story, kips
- $\alpha$ = ASD/LRFD force level adjustment factor, equal to 1.0 (LRFD) or 1.6 (ASD)
- $\Delta H$ = first-order interstory drift due to lateral forces, in.

Thus a limit on the stability coefficient, $\theta$, is a limit on geometric stability ($P$-$\Delta$) effects.

### DERIVATION OF THE INDIRECT ANALYSIS METHOD

The IAM is derived from the DM, with the simplification that stiffness reductions, rather than being applied member by member, are computed story by story and applied either story by story or globally. This approach permits quantifying the effect and applying it as a factor in an analysis using full stiffness properties rather than requiring a modified model for stability design.

Following the method presented in Design Guide 28 (Griffis and White, 2013), the IAM is derived in the following based on a dual-purpose second-order amplifier, $B_2$, that addresses both geometric nonlinearity ($P$-$\Delta$ effects) and the effects of stiffness reduction. [The $B_2$ amplifier was developed as $B_2$ in White et al. (2007)]. Note that use of this amplifier to determine displacements is equivalent to establishing a reduced stiffness due to geometric effects combined with the effects of inelasticity, member imperfection, and stiffness uncertainty. The amplification has a nonlinear relationship to the destabilizing gravity load. However, the resulting stiffness defines a linear relationship between lateral load and displacement.

The second-order shear (with notional loads) is set equal to first-order shear with additional forces required for design for stability:

$$B_2 (H + \Sigma N) = H + H PA$$

where

- $B_2$ = second-order amplifier using the stiffness reduction $EI^* = 0.8 \tau b EI$
- $H PA$ = $P$-$\Delta$ shear forces [from Equation B-10b in Griffis and White (2013)], kips
- $\Sigma N$ = notional loads to address out-of-plumbness (at or above the level under consideration), kips
- $E$ = modulus of elasticity, ksi
- $I$ = moment of inertia, in.$^4$
- $\tau b$ = stiffness-reduction parameter defined in AISC Specification Section C2, Equations C2-2a and C2-2b

The AISC Specification definition of $\tau b$ can be expressed as:

$$\tau b = 4 \frac{\alpha P_{PNS}}{P_{ns}} \left(1 - \frac{\alpha P_{PNS}}{P_{ns}}\right) \leq 1.0$$

where

- $P_{PNS}$ = cross-section compressive strength, kips
- $P_{ns} = F_y A_g$ (for columns without slender elements), kips
- $F_y A_g$ (for columns with slender elements), kips
- $P_{ns} =$ required axial compressive strength using LRFD or ASD load combinations, kips
- $A_g =$ effective area, in.$^2$
- $A_g =$ gross area of member, in.$^2$
- $F_y =$ specified minimum yield stress, ksi

Griffis and White (2013) use an equality similar to Equation 2 to establish that FOM forces correspond to DM forces within the limits of FOM application. The displacement-magnification term $B_2$ in the Design Guide 28 formulation [Equation B-4 in Griffis and White (2013)] incorporates a stiffness-reduction factor of 0.8 for the FOM:

$$B_2 = \frac{1}{1 - \frac{Q}{0.8}}$$

where $Q$ is a stability index, related to the stability coefficient $\theta$; thus:
\[ Q = \theta / R_M \]  

and \( R_M \) is a coefficient to account for member \( P-\delta \) influence on structure \( P-\Delta \).

The factor of 0.8 in Equation 4 represents the stiffness reduction of the DM, with the limitation that in the FOM, axial stress may not be permitted to reach a level sufficient to affect flexural stiffness of moment-frame beams and columns. A more general form of Equation 4 can be written to incorporate the axial-force-dependent flexural-stiffness-reduction parameter, \( \tau_b \):

\[ \overline{B}_2 = \frac{1}{1 - \frac{Q}{0.8\tau_b}} = \frac{1}{1 - \frac{\theta}{0.8\tau_b}R_M} \]  

Incorporation of \( \tau_b \) into \( \overline{B}_2 \) effectively applies this parameter uniformly to all members and all actions (axial as well as flexure). This extensive application of \( \tau_b \) to the entire lateral load effect potentially overestimates its influence but permits the derivation of a single factor on the lateral load to account for the effect of high axial stress on the system stiffness.

The stability index, \( Q \), can also be expressed thus:

\[ Q = \frac{\alpha P_{story} \Delta H}{R_M HL} \]  

The stability index \( Q \) is thus more complete than the stability coefficient \( \theta \), including the stiffness reduction related to moment-frame column compressive force, \( R_M \). The stability coefficient, \( \theta \), however, is used in the ASCE 7 standard (ASCE, 2016) and is therefore used in this paper (in conjunction with \( R_M \)). The terms that define the stability index \( Q \) in Equation 7 can also be used to define the \( B_2 \) amplifier:

\[ B_2 = \frac{1}{1 - \frac{\alpha P_{story} \Delta H}{R_M HL}} \]  

Thus, the index \( Q \) can be expressed as a function of \( B_2 \):

\[ Q = 1 - \frac{1}{B_2} \]  

For development of the IAM, the geometric effects and stiffness-reduction effects that comprise the factor \( \overline{B}_2 \) are distinguished:

\[ \overline{B}_2 = B_2 B_3 \]  

where

\[ B_3 = \text{IAM amplification factor to account for stiffness reduction due to inelasticity, member imperfection, and stiffness uncertainty} \]

Thus, \( B_2 \) addresses geometric second-order effects, and \( B_3 \) addresses stiffness reduction due to inelasticity, member imperfection, and stiffness uncertainty. This separation permits the use of typical second-order analysis using nominal stiffness properties with a separate factor, \( B_3 \), to account for the effects of the stiffness reduction inherent in the DM. By separating \( B_2 \) and \( B_3 \), the IAM does not affect established second-order analysis procedures that use full-stiffness member properties, including those incorporated into analysis software. [Use of the \( B_2 \) amplifier is valid only for systems with vertical columns (AISC, 2016), and thus the IAM (which utilizes \( B_2 \)) is similarly limited.]

Combining Equation 6 and Equation 10 gives:

\[ B_2 B_3 = \frac{1}{1 - \frac{\theta}{0.8\tau_b}R_M} \]  

The second-order amplifier \( B_2 \) can also be written in similar terms (similar to Equation B-3 in Griffis and White, 2013):

\[ B_2 = \frac{1}{1 - \frac{\theta}{R_M}} \]  

Equation 11 and Equation 12 are combined to determine \( B_3 \):

\[ B_3 = \frac{1 - \frac{\theta}{0.8\tau_b}R_M}{1 - \frac{\theta}{R_M}} \]  

The factor \( B_3 \) is a function of the stability coefficient, \( \theta \), the \( P-\delta \) factor, \( R_M \), and the flexural-stiffness-reduction parameter, \( \tau_b \). Limits on these values can be set in advance of design such that the factor \( B_3 \) may be determined and incorporated into the analysis from the first iteration. \( B_3 \) is also fundamentally an expression of the 0.8 stiffness-modifier term, but this parameter is fixed.

Equation 13 can be written in terms of \( B_2 \) and \( \tau_b \) utilizing Equation 9 and Equation 5:

\[ B_3 = \frac{0.8\tau_b}{1 - (1 - 0.8\tau_b)B_2} \]  

Equation 13 or 14 may be used to amplify the results of a second-order analysis. If \( \alpha P_{xx}/P_{xx} \leq 0.5 \), \( \tau_b = 1.0 \), and Equation 14 reduces to:

\[ B_3 = \frac{4}{5 - B_2} \]  

**USE OF THE SECOND-ORDER EFFECT DETERMINED FROM ANALYSIS**

If an explicit second-order analysis is performed, a second-order amplification term may be determined from the analysis results and used in lieu of \( B_2 \) from AISC Specification Equation A8-6 to calculate the stiffness-reduction amplifier.
This amplification term (shown here as \( B'_3 \)) is determined using the ratio of second-order displacements to first-order displacements from the analysis:

\[
B'_3 = \frac{\Delta_2}{\Delta_1}
\]  

(16)

where

\( \Delta_1 \) = first-order story drift using full stiffness properties, in.

\( \Delta_2 \) = second-order story drift using full stiffness properties, in.

The corresponding stiffness-reduction amplifier, \( B'_2 \), is obtained by combining Equation 14 and Equation 16 (with \( B'_2 = B'_2 \)):

\[
B'_2 = \frac{0.8 \tau_b}{1 - [1 - 0.8 \tau_b] \frac{\Delta_2}{\Delta_1}}
\]  

(17)

**Notional Loads**

The IAM notional loads are the same as are required for the DM. These notional loads are:

\[
N_i = \frac{\Delta_{no}}{L} Y_i
\]  

(18)

where

\( N_i \) = notional load at level \( i \), kips

\( Y_i \) = gravity load applied at level \( i \), kips

\( \Delta_{no} \) = nominal initial out-of-plumbness, in.

The initial out-of-plumbness, \( \Delta_{no} \), is taken as 0.002 times the story height in the AISC Specification (AISC, 2016). Notional loads are applied story by story, and thus accumulate as the vertical load supported, \( P_{story} \), accumulates.

Consistent with the DM, these notional loads are proposed to be applied for gravity-only load combinations for cases with \( B_2 \leq 1.5 \) and for all load combinations with \( B_2 > 1.5 \). This results in a minor error for load combinations that include lateral loads and have \( B_2 \leq 1.5 \); this too is consistent with the DM.

As the IAM has been derived to be consistent with the DM, the alternative of using an additional notional load equal to 0.001 times the gravity load and \( \tau_b = 1.0 \) in lieu of \( \tau_b < 1.0 \) may be used [AISC Specification, C2.3(c)].

**Values of \( B_3 \) and Proposed Range of Applicability of the Indirect Analysis Method**

Table 1 shows the factor \( B_3 \) for different combinations of \( B_2 \) and the flexural stiffness-reduction parameter, \( \tau_b \), using Equation 14. Values of \( B_3 \) are presented corresponding to the values of \( B_2 \) ranging from 1.00 to 5.00 and \( \alpha P_r/P_{ns} \) ranging from 0 to 1.00. \( B_2 = 1.0 \) corresponds to a system that has no gravity load (or is infinitely stiff). \( B_2 = 5.00 \) and \( \alpha P_r/P_{ns} = 1.00 \) conditions are unstable regardless of other factors. Shading indicates bands of similar values. Linear interpolation may be used with this table to obtain a liberal estimate of \( B_3 \).

The \( B_3 \) amplifier increases disproportionately with increasing axial-load ratios \( \alpha P_r/P_{ns} \) and second-order amplifier \( B_2 \). At higher ranges of these parameters, the calculation results in negative or zero values, indicating insufficient stiffness to prevent instability regardless of strength; these cases are shown in black. High values of \( B_3 \) indicate high sensitivity to loading and modeling assumptions. In practice, a limit on the permissible \( B_3 \) value may be used to prevent large sensitivity to small approximations, errors, or uncertainties in loading or modeling.

The IAM is not proposed for \( B_2 > 2.0 \) due to this sensitivity. For \( B_2 > 1.5 \), a second-order analysis that includes the effect of \( P-\delta \) on \( P-\Delta \) is necessary (Nair, 2009). (While use of a \( B_2 \) amplifier that includes \( R_M \) complies with this requirement, an explicit second-order analysis reduces error.) Similarly, the IAM is not proposed for \( \alpha P_r/P_{ns} > 0.7 \) for moment-frame columns.

The proposed IAM may be used if the following conditions are met:

- \( \alpha P_r/P_{ns} \leq 0.7 \) for moment-frame columns.
- Columns are vertical.
- A second-order analysis is performed (or the factors \( B_1 \) and \( B_2 \) are applied).
- Notional loads are applied for \( B_2 \geq 1.5 \) and for gravity-only load combinations.
- \( B_3 \) is taken as the largest value at or above the level under consideration.
- The value of \( \tau_b \) is determined (or confirmed) using the results of a second-order analysis that is amplified using the factor \( B_3 \).
- The largest displacement of the story is used to determine both \( B_2 \) and \( B_3 \).

For torsional loading (i.e., lateral loading that causes the diaphragm to rotate), Equations 16 and 17 may be used with results from a three-dimensional analysis, or values of \( B_2 \) and \( B_3 \) may be computed based on maximum drifts. Notional load patterns should include a torsion-inducing pattern.

**FIRST-ORDER ANALYSIS**

Engineers accustomed to performing a first-order analysis may prefer to use amplifiers to address both second-order effects (\( B_2 \)) and the stiffness-reduction (\( B_3 \)). This product,
obtained by multiplying Equation 14 by \( B_2 \), may be applied to the lateral loads or to their effects determined using a first-order analysis:

\[
B_2B_3 = \frac{0.8\tau_b}{\sqrt{B_2} - (1 - 0.8\tau_b)}
\]  

(19)

This product is also a useful indication of the magnitude of stability effects on the design. Such an approach is a version of the IAM (rather than the FOM), and the limits discussed in the previous section apply. If \( \alpha P_r / P_{ns} \leq 0.5 \), \( \tau_b = 1.0 \), and Equation 19 reduces to:

\[
B_2B_3 = \frac{4}{\sqrt{B_2} - 1}
\]  

(20)

This equation can be used as an amplification factor on lateral loads with greater economy compared to the FOM, with results corresponding to those that would be obtained from the DM. At \( B_2 = 1.5 \) this equation results in \( B_2B_3 = 1.71 \), and for gravity-only load combinations, the amplified notional load \( (B_2B_3N) = 1.71 \times 0.002P_{r,story} = 0.00343P_{r,story} \), 82% of what is required by the FOM in the AISC Specifications. [This difference is due to the conservative amplification of the initial out-of-plumbness, \( \Delta_0 \), in the FOM by dividing by 0.8, which simplifies the FOM derivation (Equation B-9b in Griffis and White, 2013).] The advantage provided by the IAM is greater at lower values of \( B_2 \) because the FOM forces are based on \( B_2 = 1.5 \) regardless of the actual second-order effect.

Table 2 shows the product of the factors \( B_2 \) and 3 for different combinations of \( B_2 \) and flexural stiffness-reduction parameter, \( \tau_b \). The table utilizes Equation 19 \((B_2B_3)\). Similar to Table 1, shading in Table 2 marks bands of similar values. Linear interpolation may be used with this table to obtain a liberal estimate of the product \( B_2B_3 \).

### APPLICATION

Similar to the DM, the IAM addresses the stiffness-reduction effects in the analysis rather than in the design strength equations (as the ELM does). As discussed in the derivation, the \( B_2 \) and \( B_3 \) amplifiers represent a reduced linear lateral stiffness. The IAM amplification may thus be applied to loads prior to the second-order analysis or to their effects afterward, prior to design, or (conservatively) to demand-to-capacity ratios. Simpler methods of application are necessarily more conservative. Methods of application include:

- **Demand-to-capacity ratio limit.** The simplest method is to use the inverse of the factor \( B_3 \) as a limit on permissible demand-to-capacity ratios. This method effectively amplifies the gravity load effect, which introduces arbitrary conservatism; this may be acceptable for cases where \( B_3 \) is small. For example, in seismic design per ASCE/SEI 7 (ASCE, 2016), limits on the stability coefficient, \( \theta \), for special moment frames are such that if \( \alpha P_r / P_{ns} \leq 0.5 \), the product \( B_2B_3 \) cannot exceed 1.15; for this value, demand-to-capacity ratios not exceeding 0.866 for a first-order analysis are acceptable without more refined amplification and combination of loads.

- **Amplification of loads.** Alternatively, the value of \( B_3 \) can be selected in advance of design by limiting \( \alpha P_r / P_{ns} \) and taking \( B_2 \) as the lesser of that corresponding to the limit on the stability coefficient, \( \theta \), and that corresponding to the drift limit. This value of \( B_3 \) may be applied as a supplementary lateral load factor. The factor \( B_3 \) may be used to amplify the entire set of results from a lateral analysis, utilizing the largest value of \( B_3 \) for all stories. Because \( B_3 \) is based on the smallest value of \( \tau_b \) for all moment-frame columns in a story, this may extend the effect of a single low value of \( \tau_b \) dramatically, but in typical practice, moment-frame columns rarely have axial force above 0.5\( P_{ns} \), and the axial force must be substantially greater than this value to have a significant effect on stiffness. If some \( \alpha P_r / P_{ns} \) ratios exceed 0.5 (and member selection is governed by strength rather than the stiffness required to meet drift limits), story-by-story application of \( B_3 \) may be preferable.

- **Story-by-story amplification of load effects.** Use of a single value of \( B_3 \) on the entire lateral load will be conservative for stories where the second-order effect is relatively low. \( B_3 \) may be applied to the load effect story by story instead of as a global factor. However, use of the \( B_3 \) factor as a story-specific factor on the applied force (rather than on the load effect) is potentially unconservative because the amplification of overturning effects is less than the shear amplification at levels with a higher value of \( B_3 \) than the levels above (which is typical if drifts are uniform), and thus this method is not recommended until this potential discrepancy is studied. Story-by-story application of the product \( B_2B_3 \) on the lateral load effect is straightforward with superposition of load effects from a first-order analysis. Story-by-story application of factor \( B_3 \) with the results of a second-order analysis may be done by developing a model with geometric stiffness based on the vertical loads; analyses using this geometric-stiffness model are suitable for linear superposition (Wilson and Habibullah, 1987) and are thus suitable for application of the \( B_3 \) factor either to lateral loads or lateral-load effects.

The proposed procedure for IAM application is:

1. Establish limit on drift and corresponding vertical and lateral loads.
2. Establish vertical and lateral loads for strength evaluations.
3. Size members to meet drift limit.

4. Check stability coefficient limit $\theta$, if required (as it is for seismic design).

5. Determine second-order effect $B_2$ (Equation 12) for the applicable load combinations.

6. Calculate preliminary value of $B_3$ (based on $\alpha P_r / P_{ns} \leq 0.5$ using Equation 15).

7. Apply additional notional load of $0.001P_{story}$ to preclude the need to check $\alpha P_r / P_{ns}$. (This is optional; engineers may consider the check on axial load ratio to be less cumbersome than implementing the notional load. Also, this notional-load approach may result in a more conservative evaluation.)

8. Perform second-order analysis with lateral loads amplified by the largest value of $B_3$.

### Table 1. Amplification Factors $B_3$ for IAM

<table>
<thead>
<tr>
<th>$\frac{\alpha P_r}{P_{ns}}$</th>
<th>$\leq 0.5$</th>
<th>0.55</th>
<th>0.6</th>
<th>0.65</th>
<th>0.7</th>
<th>0.75</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>1</td>
<td>0.99</td>
<td>0.96</td>
<td>0.91</td>
<td>0.84</td>
<td>0.75</td>
<td>0.64</td>
<td>0.51</td>
<td>0.36</td>
<td>0.19</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$B_2$</th>
<th>$B_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>1.05</td>
<td>1.01</td>
</tr>
<tr>
<td>1.10</td>
<td>1.03</td>
</tr>
<tr>
<td>1.15</td>
<td>1.04</td>
</tr>
<tr>
<td>1.20</td>
<td>1.05</td>
</tr>
<tr>
<td>1.25</td>
<td>1.07</td>
</tr>
<tr>
<td>1.30</td>
<td>1.08</td>
</tr>
<tr>
<td>1.35</td>
<td>1.10</td>
</tr>
<tr>
<td>1.40</td>
<td>1.11</td>
</tr>
<tr>
<td>1.45</td>
<td>1.13</td>
</tr>
<tr>
<td>1.50</td>
<td>1.14</td>
</tr>
<tr>
<td>1.55</td>
<td>1.16</td>
</tr>
<tr>
<td>1.60</td>
<td>1.18</td>
</tr>
<tr>
<td>1.65</td>
<td>1.19</td>
</tr>
<tr>
<td>1.70</td>
<td>1.21</td>
</tr>
<tr>
<td>1.75</td>
<td>1.23</td>
</tr>
<tr>
<td>1.80</td>
<td>1.25</td>
</tr>
<tr>
<td>1.85</td>
<td>1.27</td>
</tr>
<tr>
<td>1.90</td>
<td>1.29</td>
</tr>
<tr>
<td>1.95</td>
<td>1.31</td>
</tr>
<tr>
<td>2.00</td>
<td>1.33</td>
</tr>
<tr>
<td>2.20</td>
<td>1.43</td>
</tr>
<tr>
<td>2.40</td>
<td>1.54</td>
</tr>
<tr>
<td>2.60</td>
<td>1.67</td>
</tr>
<tr>
<td>2.80</td>
<td>1.82</td>
</tr>
<tr>
<td>3.00</td>
<td>2.00</td>
</tr>
<tr>
<td>3.50</td>
<td>2.67</td>
</tr>
<tr>
<td>4.00</td>
<td>4.00</td>
</tr>
<tr>
<td>4.50</td>
<td>8.00</td>
</tr>
<tr>
<td>5.00</td>
<td></td>
</tr>
</tbody>
</table>

Unstable
9. Verify that member demand-to-capacity ratios do not exceed 1.0. If $\alpha P_r/P_{ns} \leq 0.5$ was assumed (see step 7), check this ratio.

10. If the strength or stability checks indicate insufficiency, iterate using IAM or switch to DM. For IAM iteration, check $\alpha P_r/P_{ns}$ (to determine if the additional notional load can be removed), recalculate $B_3$, consider applying separate $B_3$ factors at each story, or resize members.

Steps 4, 5, and 6 may be done in advance of design by assuming a stiffness necessary to meet drift limits (including second-order effects), simplifying the design process. (The design example in Appendix C follows this procedure.)

Because many moment frames have member sizes selected to meet drift limits rather than for strength, iteration in the subsequent design for stability is not always required, and the IAM provides a relatively simple method of verifying adequate strength to maintain stability. If iteration is required and the DM is used, the initial IAM analysis provides the necessary information to determine $\tau_b$.

### Table 2. Amplification Factors $B_2B_3$ for IAM

<table>
<thead>
<tr>
<th>$\alpha P_r/P_{ns}$</th>
<th>$0.5$</th>
<th>$0.55$</th>
<th>$0.6$</th>
<th>$0.65$</th>
<th>$0.7$</th>
<th>$0.75$</th>
<th>$0.8$</th>
<th>$0.85$</th>
<th>$0.9$</th>
<th>$0.95$</th>
<th>$1.00$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_0$</td>
<td>1.00</td>
<td>0.99</td>
<td>0.96</td>
<td>0.91</td>
<td>0.84</td>
<td>0.75</td>
<td>0.64</td>
<td>0.51</td>
<td>0.36</td>
<td>0.19</td>
<td>0.00</td>
</tr>
<tr>
<td>$B_2$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$B_3$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

- **Unstable**
Table 3. Comparison of Methods of Design for Stability

<table>
<thead>
<tr>
<th></th>
<th>Advantages of IAM</th>
<th>Disadvantages of IAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOM</td>
<td>• Lower forces.</td>
<td>• Requires second-order analysis. However, the FOM is limited to $B_2 \leq 1.5$ and thus requires determination of the second-order effect to permit use.</td>
</tr>
<tr>
<td></td>
<td>• Permits $\alpha P_t/P_{ns} \leq 0.7$ instead of $\alpha P_t/P_{ns} \leq 0.5$ for moment frame columns.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Permits $B_2 \leq 2.0$ instead of $B_2 \leq 1.5$.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Eliminates the need for additional lateral loads (for $B_2 \leq 1.5$).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• $B_2B_3$ is an indicator of the sensitivity to modeling and loading assumptions.</td>
<td></td>
</tr>
<tr>
<td>ELM</td>
<td>• Lower demand-to-capacity ratios.</td>
<td>• Requires application of amplification factor $B_3$.</td>
</tr>
<tr>
<td></td>
<td>• $K = 1$ for moment-frame columns with $B_2 &gt; 1.1$.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Permits $B_2 \leq 2.0$ instead of $B_2 \leq 1.5$.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Provides appropriate design forces for beams and connections (instead of addressing only column strength).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• $B_3$ is an indicator of the sensitivity to modeling and loading assumptions.</td>
<td></td>
</tr>
<tr>
<td>DM</td>
<td>• Single calculation of $B_3$ per level in lieu of member-by-member application of $\tau_b$.</td>
<td>• Potentially higher forces, especially for torsional response and flexible diaphragms.</td>
</tr>
<tr>
<td></td>
<td>• Does not require one model for strength and a second model for drift and period determination.</td>
<td>• Requires use of largest $\tau_b$ for each level of moment frames if $\alpha P_t/P_{ns} &gt; 0.5$.</td>
</tr>
<tr>
<td></td>
<td>• $B_3$ is an indicator of the sensitivity to modeling and loading assumptions.</td>
<td>• Suitability limited to:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>◦ Vertical columns.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>◦ $B_2 \leq 2.0$.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>◦ $\alpha P_t/P_{ns} \leq 0.7$ for moment frame columns.</td>
</tr>
</tbody>
</table>

For mixed systems (moment frames with braced frames or shearwalls), $R_M$ determined using AISC Specification Equation A-8-8 is appropriate to use in calculating $B_3$. The axial-force ratio $\alpha P_t/P_{ns}$ is only relevant for members whose flexural resistance contributes to the lateral stiffness; braces, beams, and columns modeled with both ends pinned may have $\alpha P_t/P_{ns} > 0.5$ and not be considered in determining the value of $\tau_b$ to be used to compute $B_3$.

**VERIFICATION**

Verification studies are presented in Appendices A, B, and C. Appendix A applies the IAM to a simple frame that was studied by Carter and Geschwindner (2008) using FOM, ELM, and DM methods. That study demonstrated the advantage of the DM over the ELM and FOM in terms of lower demand-to-capacity ratios, and Appendix A shows a perfect correspondence between the IAM and the DM. Appendix B is a multi-parameter study of similar frames, comparing IAM and DM forces and design demand-to-capacity ratios. This comparison indicates an excellent correlation between design checks using DM and IAM forces over a broad range of the second-order effect ($B_2$), column slenderness, and stiffness-reduction parameter, $\tau_b$. Appendix C is a design example of a two-bay, eight-story frame, demonstrating application of the method in the design process.

**QUALITATIVE COMPARISON OF THE PROPOSED INDIRECT ANALYSIS METHOD WITH OTHER METHODS**

Table 3 shows qualitative advantages and disadvantages of the proposed IAM with respect to the established stability design methods (DM, ELM, and FOM).

**CONCLUSIONS**

The proposed indirect analysis method (IAM) is a stability design method based on the DM and developed to reduce the effort required of the designer. Calculation of the factors utilized in the IAM ($B_2$ and $B_3$) may be performed in advance of design and can be used to identify cases in which a more rigorous method of analysis and design for stability is appropriate. IAM is proposed to be limited to $B_2 \leq 2.0$ and $\alpha P_t/P_{ns} \leq 0.7$. Above these limits, the structure is highly sensitive to second-order effects and stiffness reduction (and thus to the accuracy of loading assumptions, modeling, and analysis).

The IAM has been compared to the DM for a range of second-order effects and axial-force ratios. Comparisons indicate close correlation, with the IAM becoming less conservative for cases with high column slenderness and high $P-\Delta$ effects ($B_2$). Within the proposed range of applicability,
the IAM approximates the DM very closely, with differences in demand-to-capacity ratio no greater than 1%.

The IAM is particularly well suited for designs governed by drift or stability-coefficient limits (such as for seismic design), allowing a simple confirmation of compliance with AISC stability-design requirements with a single factor. Thus, drift design and strength design, which are two distinct analyses, often with different loadings, may be performed using a single model without changing member properties for steel structures. For strength-governed designs, the IAM entails a modest degree of conservatism, which can be reduced by a story-by-story application of factors on the lateral-load effect.

The IAM can be used as the basis for an improved FOM with less conservatism and a better indication of the magnitude of stability-related effects on the design.

The IAM matches the DM for structures that have moment-frame columns with an axial-load ratio of \( \alpha \frac{P}{P_{ns}} \leq 0.5 \); it is conservative compared to the DM for structures that have moment frame columns with differing levels of axial-load ratio \( \alpha \frac{P}{P_{ns}} \) above 0.5. The level of conservatism of the IAM has not been established for more complex structures, although if member selection is governed by drift, the level of conservatism in the strength evaluation is inconsequential.

ACKNOWLEDGMENTS

Any reader of this paper will note its debt to the work of Griffis and White. Additionally, Larry Griffis provided valuable guidance in the development of this method. Matt Eatherton was a careful early reviewer who identified that this method is not limited to seismic applications. Lou Geschwindner provided a detailed review that helped clarify how this method differs from the other stability design methods. Ron Ziemian guided the use of more exact solutions incorporated into the verification study. Larry Kruth and Leigh Arber provided encouragement and helped focus this work on providing a benefit to practicing engineers. My colleagues Ozgur Atlayan, Joseph Dowd, and Susendar Muthukumar provided detailed review and assisted in the verification and application.

ABBREVIATIONS

| ASD      | Allowable strength design per AISC Specification Chapter B |
| DCR     | Demand-to-capacity ratio |
| DM      | Direct analysis method of design for stability per AISC Specification Chapter C |
| ELM     | Effective-length method of design for stability per AISC Specification Appendix 7 |
| IAM     | Indirect analysis method of design for stability |
| FOM     | First-order analysis method of design for stability per AISC Specification Appendix 7 |
| LRFD    | Load and resistance factor design per AISC Specification Chapter B |

SYMBOLS

\( A_e \)  Effective area, in.\(^2\) (AISC Specification)
\( A_g \)  Gross area of member, in.\(^2\) (AISC Specification)
\( B_2 \)  Multiplier to account for \( P-\Delta \) effects (AISC Specification Appendix 8)
\( B_2' \)  Second-order amplifier using the stiffness reduction \( EI^* = 0.8\eta EI \) (Design Guide 28)
\( B_3 \)  IAM amplification factor to account for stiffness reduction due to inelasticity, member imperfection, and stiffness uncertainty (Equation 14)
\( B_3' \)  IAM amplification factor to account for stiffness reduction due to inelasticity, member imperfection, and stiffness uncertainty calculated using \( B_2' \) (Equation 17)
\( C_L \)  Factor that accounts for the reduction in column sidesway stiffness due to the presence of axial load on the column (Design Guide 28)
\( D_{AF} \)  Displacement amplification factor (Design Guide 28)
\( D_{AF-DM} \)  Displacement amplification factor using reduced stiffness (Appendix B)
\( E \)  Modulus of elasticity, ksi (AISC Specification)
\( F_{AF} \)  Force amplification factor (Design Guide 28)
\( F_{AF-DM} \)  Force amplification factor using reduced stiffness (Appendix B)
\( F_y \)  Specified minimum yield stress, ksi (AISC Specification)
\( H \)  Total story shear, kips (AISC Specification)
\( P_{PA} \)  \( P-\Delta \) shear forces, kips (Design Guide 28)
\( I \)  Moment of inertia, in.\(^4\) (AISC Specification)
\( K \)  Effective length factor (AISC Specification Chapter E)
\( L \)  Story height, in. (AISC Specification)
\( M_{DM} \) Moment resulting from a second-order analysis using reduced stiffness as required by the direct analysis method, kip-in. (Appendix B)

\( M_{FOA} \) First-order analysis moment, kip-in. (Appendix B)

\( M_{SOA} \) Second-order analysis moment, kip-in. (Appendix B)

\( N_i \) Notional load at level \( i \), kips (AISC Specification)

\( \Sigma P \) Sum of applied vertical forces, kips, (Design Guide 28)

\( P \) Applied vertical force, kips, (Design Guide 28)

\( P_A \) Axial force in stability column, kips (Appendix B)

\( P_B \) Axial force in leaning column, kips (Appendix B)

\( P_L \) Story lateral force required to induce a first-order unit drift, kips (Design Guide 28)

\( P_{e1} \) Elastic critical buckling strength of the member in the plane of bending, kips (AISC Specification)

\( P_{nx} \) Cross-section compressive strength, kips (AISC Specification)

\( P_r \) Required axial compressive strength, kips (AISC Specification)

\( P_{story} \) Total vertical load supported by the story, kips (AISC Specification)

\( Q \) Stability index (Design Guide 28)

\( R_M \) Coefficient to account for member \( P-\delta \) influence on structure \( P-\Delta \) (AISC Specification)

\( Y_i \) Gravity load applied at level \( i \), kips (AISC Specification)

\( \alpha \) ASD/LRFD force level adjustment factor, equal to 1.0 (LRFD) or 1.6 (ASD) (AISC Specification)

\( \alpha \) Ratio of the smaller to the larger end moment in the moment-frame column, determined from a first-order sidesway analysis (Design Guide 28)

\( \beta_L \) Coefficient corresponding to the contribution of the moment-frame column to the first-order story sidesway stiffness (Design Guide 28)

\( \Delta_1 \) First-order story drift using full stiffness properties, in.

\( \Delta_2 \) Second-order story drift using full stiffness properties, in.

\( \Delta_{DM} \) Second-order story drift using reduced stiffness properties, in. (Appendix B)

\( \Delta_H \) First-order interstory drift due to lateral forces, in. (AISC Specification)

\( \Delta_o \) Nominal initial out-of-plumbness, in. (Design Guide 28)

\( \theta \) Stability coefficient for \( P-\Delta \) effects (ASCE/SEI 7)

\( \tau_o \) Flexural stiffness-reduction parameter (AISC Specification)

REFERENCES


APPENDIX A

VERIFICATION: CANTILEVER STABILITY COLUMN WITH LEANING COLUMN (EXAMPLE)

The equivalence between the IAM and the DM can be established by analyzing a well-understood example frame: the one-bay frame studied by Carter and Geschwindner (2008). The frame consists of a cantilevered stability column (left, with a rigid base connection and a pin at the top), a leaning column (right, with pins top and bottom), and a rigid beam constraining their lateral movement. This frame is shown in Figure A1. The LRFD method is employed in this example.

Carter and Geschwindner used the following values for their study:

\[ P_A = P_B = 200 \text{ kips} \]

\[ L = 15 \text{ ft} \]

\[ H = 20 \text{ kips} \]

The notional load \( N \) depends on the stability-design method employed. (No notional load is required for the IAM in this example.) The stability column is an ASTM A992 W14×90.

Carter and Geschwindner determined axial forces and moments for four different methods of design for stability:

- The effective-length method (second order).
- The first-order method.
- The direct analysis method.
- The simplified method (a version of the ELM utilizing design tables).

The IAM is applied to this frame, utilizing the results of Carter and Geschwindner’s full-stiffness second-order analysis for the system and for column A. The determination of \( B_2 \) is redone here to add a significant figure. (Such precision is not required for the IAM but is applied in this example to allow comparison of results without artifacts due to rounding.) Consistent with the 2005 edition of the AISC Specification (AISC, 2005), Carter and Geschwindner use a value of \( R_M \) of 0.85 rather than calculating \( R_m \) using AISC Specification Equation A-8-8 per the 2016 edition (AISC, 2016); for purposes of comparison of the stability-design methods, the same value is used here.

Fig. A1. Frame used for verification study.
\[
\Delta_H = \frac{HL^3}{3EI} \\
= \frac{(20 \text{ kips})(15 \text{ ft})(12 \text{ in./ft})^3}{3(29,000 \text{ ksi})(999 \text{ in.}^4)} \\
= 1.34 \text{ in.}
\]

\[
B_2 = \frac{1}{1 - \frac{\alpha P_{\text{story}}\Delta_H}{R_MHL}} \\
= \frac{1}{1 - \frac{1.0(400 \text{ kips})(1.34 \text{ in.})}{0.85(20 \text{ kips})(180 \text{ in.})}} \\
= 1.21 \text{ in.}
\]

\[
M_H = HL \\
= (20 \text{ kips})(15 \text{ ft}) \\
= 300 \text{ kip-ft}
\]

\[
P_r = P_H \\
= 200 \text{ kips}
\]

\[
\frac{P_r}{P_c} = 0.200
\]

From AISC Manual Table 3-10, the available flexural strength, \(\phi M_n\), is

\[
\phi M_n = 573 \text{ kip-ft}
\]

\[
P_{ns} = A_g F_y \\
= (26.5 \text{ in.}^2)(50 \text{ ksi}) \\
= 1,330 \text{ kips}
\]

The axial load ratio is:

\[
\frac{\alpha P_r}{P_{ns}} = \frac{(1.00)(200 \text{ kips})}{1,330 \text{ kips}} \\
= 0.15
\]

Because \(\frac{\alpha P_r}{P_{ns}} < 0.5\), \(\tau_b\) is 1.0 and the IAM amplifier is obtained from Equation 15

\[
B_3 = \frac{4}{5 - B_2} \\
= \frac{4}{5 - 1.21} \\
= 1.06
\]
The IAM moment is:

\[ M_r = B_2B_3M_l \]

\[ = (1.21)(1.06)(300 \text{ kip-ft}) \]

\[ = 385 \text{ kip-ft} \]

Because \( P_r/P_c \geq 0.2 \), AISC Specification Equation H1-1a applies:

\[ \frac{P_r}{P_c} + \frac{8}{9} \left( \frac{M_r}{\phi M_p} \right) = 0.200 + \frac{8}{9} \left( \frac{385 \text{ kip-ft}}{573 \text{ kip-ft}} \right) \]

\[ = 0.796 \]

Table A1 shows the interaction equation values from Carter and Geschwindner for the FOM, ELM, simplified ELM, and DM using AISC Specification Equation H1-1a. The table adds the IAM value computed above, which precisely matches the DM for this simple system.

### APPENDIX B

**VERIFICATION: CANTILEVER STABILITY COLUMN WITH LEANING COLUMN (PARAMETRIC STUDY)**

To verify that the proposed IAM produces forces consistent with the DM over a broad range of multiple parameters, the results of calculations similar to those in Appendix A are presented here in tabular form. These examples use a frame of the same configuration as that in Appendix A (Figure A1), analyzed using first-order analysis, second-order analysis, and DM (second-order analysis with modified stiffness properties). The stability column is an ASTM A992 W12x120 oriented in the strong axis and restrained from lateral torsional buckling along its length such that the flexural strength is based on the plastic section modulus (\( \phi M_n = \phi M_p = 9,300 \text{ kip-in.} \)). The column is restrained from minor-axis buckling at the top and bottom. The LRFD method is employed in this example.

Several cases were studied, covering a range of second-order effects and axial-force demands on the stability column and multiple frame heights. The analyses were conducted using formulae for flexural stiffness and do not include shear deformations. The method was confirmed using analysis software and benchmark problems. Using formulae permits a systematic tabular comparison of results from DM and IAM evaluations varying multiple parameters in a spreadsheet.

Following methods developed by LeMessurier (1977), second-order effects incorporate either a force-amplification factor, \( F_{AF} \), or a displacement amplification factor, \( D_{AF} \). Nomenclature and equations for these factors correspond to those used by Griffis and White (2013).

The force amplification factor is presented in Equation A-9 in Griffis and White (2013):
\[ F_{AF} = \frac{1}{1 - \frac{\Sigma P}{P_L - \Sigma C_L P}} \]  

(21)

where

- \( C_L \) = factor that accounts for the reduction in column sidesway stiffness due to the presence of axial load on the column
- \( P \) = applied vertical force, kips
- \( P_L \) = story lateral force required to induce a first-order unit drift, kips
- \( \Sigma P \) = sum of applied vertical forces, kips

In the example frame, the vertical force is:

\[ \Sigma P = P_A + P_B \]  

(22)

For a cantilever column, the parameter \( P_L \) is:

\[ P_L = \frac{3EI}{L^2} \]  

(23)

The factor \( C_L \) is zero for columns not providing lateral stiffness. For the cantilever column \( C_L \) is:

\[ C_L = \left( \frac{12}{\pi^2} - 1 \right) \left( \frac{\beta_L}{3 + 9\alpha} \right)^2 \]  

(24)

where

- \( \alpha \) = ratio of the smaller to the larger end moment in the moment-frame column, determined from a first-order sidesway analysis
- \( \beta_L \) = coefficient corresponding to the contribution of the moment-frame column to the first-order story sidesway stiffness
  - \( \beta_L = 3 \) for a cantilever column

Thus, for the frame under consideration:

\[ F_{AF} = \frac{1}{1 - \frac{P_A + P_B}{\frac{3EI}{L^2} - \left( \frac{12}{\pi^2} - 1 \right) P_A}} \]  

(25)

For the DM, the stiffness term is reduced by 0.8\( \tau_b \):

\[ F_{AF-DM} = \frac{1}{1 - \frac{P_A + P_B}{0.8\tau_b \frac{3EI}{L^2} - \left( \frac{12}{\pi^2} - 1 \right) P_A}} \]  

(26)

The displacement amplification factor \( D_{AF} \) is presented in Equation A-10 in Griffis and White (2013):

\[ D_{AF} = \frac{1}{1 - \frac{\Sigma P + \Sigma C_L P}{P_L}} \]  

(27)
For the frame under consideration:

\[
D_{AF} = \frac{1}{12 \frac{P_A + P_B}{E I} \left(1 - \frac{P_A + P_B}{E I L^2} \right)}
\]  

(28)

For the DM the stiffness term is reduced by 0.8\( \tau_b \):

\[
D_{AF-DM} = \frac{1}{12 \frac{P_A + P_B}{E I} \left(1 - \frac{0.8 \tau_b P_A + P_B}{E I L^2} \right)}
\]  

(29)

Moments are determined as follows:

\[
M_{FOA} = HL
\]  

(30)

\[
M_{SOA} = F_{AF} HL
\]  

(31)

\[
M_{DM} = F_{AF-DM} HL
\]  

(32)

Displacements are determined as follows:

\[
\Delta_1 = \frac{HL^3}{3EI}
\]  

(33)

\[
\Delta_2 = D_{AF} \Delta_1
\]  

(34)

\[
\Delta_{DM} = D_{AF-DM} \Delta_1
\]  

(35)

Table B1 provides a systematic comparison of DM and IAM demand-to-capacity ratios for varying column height, axial load ratios \(\alpha P_i / P_{ns} \), and second-order effects \(B_2 \). The table shows the analysis and loading information and calculated \( \tau_b \) factors for the stability column, as well as notional loads where there is no applied lateral load and where the \(B_2\) factor is greater than 1.5. The table also shows the results, including analyzed first-order and second-order displacement \(\Delta_1\) and \(\Delta_2\), the second-order displacement corresponding to the DM \(\Delta_{DM}\), and analyzed first-order, second-order, and DM moments \(M_{FOA}, M_{SOA}, M_{DM}\).

Table B1 shows factors \(B_2\) and \(B_3\) determined in two ways. First, \(B_2\) is calculated using Equation 12, and the corresponding value of \(B_3\) is determined using Equation 14. Second, a value of \(B_2^*\) is calculated as the ratio of \(\Delta_2\) to \(\Delta_1\) from the analysis (Equation 16), and the corresponding value of \(B_3^*\) is determined (Equation 17). The table presents the \(F_{AF}\) and \(D_{AF}\) factors (Equations 25–29), and the amplifier products \(B_2 B_3\) and \(B_2^* B_3^*\) for comparison.

The table shows calculated demand-to-capacity ratios for the stability column using Chapter H interaction equations from the AISC Specification for DM, IAM (using \(B_3\)), and IAM (using \(B_2^*\)) forces. Frame heights were selected to provide a range of stability column slenderness. Gravity loads were selected to achieve target values of \(B_2\), and to provide both high- and low-axial-stress cases. Lateral loads were selected such that the DM produced demand-to-capacity ratios of 1.00 (as shown in the DCR column).

Cases B1–B4 [at the top of the table) are the results from Benchmark Problem Case 2 in AISC Specification Figure C-C2.3 (AISC, 2016), neglecting shear deformations. Results confirm that accurate moments and displacements are obtained using the formulae in the table.

Cases 1–6 cover a range of \(B_2\) from 1.11 to 1.61 with high axial-force in the stability column such that \(\alpha P_i / P_{ns} = 0.7\) and \(\tau_b = 0.84\). (The very low slenderness permits such high force.) The stability column has \(P_A / P_{e1} = 0.10\) (where \(P_{e1}\) is the Euler buckling load
determined in the plane of bending per AISC Specification Equation A-8-5). The system reaches the “limit of stability” (i.e., a demand-to-capacity ratio of 1.0 under vertical and notional loads alone with no applied lateral force) at $B_2 = 1.61$.

Cases 7–17 cover a range of $B_2$ from 1.10 to 3.00 with no axial force in the stability column, which has low slenderness.

Cases 18–20 cover a range of $B_2$ from 1.5 to 1.82 with high axial force in the low-slenderness stability column, which has $P_A/P_{ed} = 0.89$. Because of the high stability-column axial force and column height, lower values of $B_2$ cannot be achieved. The stability column has $P_A/P_{ed} = 0.34$. The system reaches the limit of stability at $B_2 = 1.82$.

Cases 21–28 cover a range of $B_2$ from 1.40 to 3.00 with moderate axial force in the moderate-slenderness stability column, which has $P_A/P_{ed} = 0.29$.

Cases 29–38 are similar to cases 21–28 and cover a range of $B_2$ from 1.10 to 3.00. This range of $B_2$ is achieved by varying the stability-column axial force rather than the leaning-column axial force.

Cases 39–44 are similar to case 34. Both $P_A$ and $P_B$ are varied such that $B_2 = 2.00$ is maintained for a range of $P_A/P_{ed}$ from 0.09 to 0.52.

Cases 45–50 vary both $P_A$ and $P_B$ such that $B_2 = 2.00$ is maintained for a range of slenderness ($L/r$) from 64 to 383.

Table B1 can be used to assess the correspondence of the IAM to the DM using both the design moments obtained and the design interaction value from AISC Specification Section H1.1 (AISC, 2016). Comparison of the moments provides a measure of the accuracy of the IAM. However, in cases where the axial load in the stability column is large, the moment represents a small contribution to the design interaction ratio, and thus differences in moment are of less consequence. Demand-to-capacity ratios differing from 1.00 are highlighted in the table, as are ratios of amplified moments to DM moments differing from 1.00.

### Comparison of Moments

The table presents three moment ratios:

- Amplified first-order moment to DM moment ($B_2 B_3 M_{FOA}/M_{DM}$).
- Amplified second-order moment to DM moment ($B_3 M_{SOA}/M_{DM}$).
- Amplified second-order moment to DM moment ($B_3^2 M_{SOA}/M_{DM}$) based on analyzed displacements.

Ratios above 1.00 indicate that the IAM is conservative, while ratios below 1.00 indicate the opposite. The results indicate that use of the IAM with amplifiers $B_2^2$ and $B_3^2$ based on analyzed second-order displacements is always conservative in the range studied. Use of first-order analysis with amplifiers $B_2$ and $B_3$ based on first-order displacements is conservative for $B_2 \leq 2.5$. Second-order analysis with the amplifier $B_3$ based on first-order displacements is conservative for $B_2 \leq 2.0$. For cases with $B_2 \geq 2.0$ and $P_A/P_{ed} \geq 0.29$, use of the amplifier $B_3$ based on first-order displacements becomes slightly unconservative. As noted by Surovek-Maleck and White (2004), the approximate methods of second-order analysis may become less reliable as column $P_A/P_{ed}$ increases.

### Comparison of Interaction Ratios

Comparison of interaction ratios was limited to IAM using second-order analysis; amplified first-order analysis forces were not evaluated. Interaction ratios were evaluated for the DM, the IAM, and the IAM’ (using the amplifiers $B_2'$ based on analyzed second-order displacements). (Horizontal forces were selected to produce DM demand-to-capacity ratios of 1.00.) Cases in which the IAM demand-to-capacity ratio is greater than 1.00 indicate that the IAM is conservative compared to the DM; cases in which the IAM demand-to-capacity ratio is less than 1.00 indicate that the IAM is unconservative. Cases in which the IAM produces an unconservative demand-to-capacity ratio by more than 0.005 are outside the range of proposed applicability. In short, the IAM matches the DM or is slightly conservative for $B_2 \leq 2.0$. Above that range, $B_3$ (which is a function of $B_2$) could be unconservative as $B_2$ deviates slightly from $\Delta_2/\Delta_1$. Based on the results, use of the IAM is conservative for $B_2 \leq 2.0$ and use of the IAM’ (which utilizes $B_2'$ from Equation 17) is always conservative in the range studied.
### Table B1. Verification Analysis Summary

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>L (in.)</td>
<td>Lc</td>
<td>H (kip)</td>
<td>N (kip)</td>
<td>Px (kip)</td>
<td>e</td>
<td>Ppx/Pv</td>
<td>Pyy/Pv</td>
<td>Pxy/Pv</td>
<td>Rxx</td>
<td>Bx</td>
<td>Bxx</td>
<td>Fxx</td>
<td>Fxx-OM</td>
<td>Dxx</td>
<td>Dxx-OM</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>20</td>
<td>2.5</td>
<td>0.1</td>
<td>62</td>
<td>0.85</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>40</td>
<td>5.0</td>
<td>0.2</td>
<td>124</td>
<td>1.63</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>60</td>
<td>7.5</td>
<td>0.3</td>
<td>186</td>
<td>2.41</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>80</td>
<td>10.0</td>
<td>0.4</td>
<td>248</td>
<td>3.19</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>100</td>
<td>12.5</td>
<td>0.5</td>
<td>310</td>
<td>3.97</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>6000</td>
<td>120</td>
<td>15.0</td>
<td>0.6</td>
<td>372</td>
<td>4.75</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>7000</td>
<td>140</td>
<td>17.5</td>
<td>0.7</td>
<td>434</td>
<td>5.53</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

Notes:

1. B1 336 176 1.0 0 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
2. B2 336 176 1.0 0 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
3. B4 336 176 1.0 0 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

126 / ENGINEERING JOURNAL / SECOND QUARTER / 2020
### Table B1. Verification Analysis Summary (continued)

<table>
<thead>
<tr>
<th>$\Delta$ (in)</th>
<th>$d$ (in)</th>
<th>$d_{out}$ (in)</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_1$,$B_2$</th>
<th>$M_{out}$</th>
<th>$M_{out}$</th>
<th>$M_{out}$</th>
<th>$\Delta_{IR}$</th>
<th>$\Delta_{IM}$</th>
<th>$B_{IR}M_{out}$</th>
<th>$B_{IM}M_{out}$</th>
<th>$B_{IR}M_{out}$</th>
<th>Notes</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90</td>
<td>1.00</td>
<td>1.34</td>
<td>636</td>
<td>636</td>
<td>636</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B1</td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B2</td>
</tr>
<tr>
<td>1.15</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B3</td>
</tr>
<tr>
<td>1.20</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B4</td>
</tr>
<tr>
<td>1.30</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B5</td>
</tr>
<tr>
<td>1.40</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B6</td>
</tr>
<tr>
<td>1.50</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B7</td>
</tr>
<tr>
<td>1.60</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B8</td>
</tr>
<tr>
<td>1.70</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B9</td>
</tr>
<tr>
<td>1.80</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B10</td>
</tr>
<tr>
<td>1.90</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B11</td>
</tr>
<tr>
<td>2.00</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B12</td>
</tr>
<tr>
<td>2.10</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B13</td>
</tr>
<tr>
<td>2.20</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B14</td>
</tr>
<tr>
<td>2.30</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B15</td>
</tr>
<tr>
<td>2.40</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B16</td>
</tr>
<tr>
<td>2.50</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B17</td>
</tr>
<tr>
<td>2.60</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B18</td>
</tr>
<tr>
<td>2.70</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B19</td>
</tr>
<tr>
<td>2.80</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B20</td>
</tr>
<tr>
<td>2.90</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B21</td>
</tr>
<tr>
<td>3.00</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B22</td>
</tr>
<tr>
<td>3.10</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B23</td>
</tr>
<tr>
<td>3.20</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B24</td>
</tr>
<tr>
<td>3.30</td>
<td>1.00</td>
<td>1.00</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td>336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B25</td>
</tr>
</tbody>
</table>

**Notes:**
- DM: Design Method
- IAM: Improved Analysis Method
- IAP: Improved Analysis Procedure
- MFOA: More Flexible Optimization Algorithm
- MIR: More Improved Analysis Method
- MIP: More Improved Analysis Procedure
- B: Baseline
- BIR: Baseline Improved
- BIM: Baseline Improved Method
- BIRM: Baseline Improved Method and Procedure

**Case:**
- B1: Baseline
- B2: Baseline Improved
- B3: Baseline Improved Method
- B4: Baseline Improved Method and Procedure
- B5: Baseline Improved Method and Procedure
- B6: Baseline Improved Method and Procedure
- B7: Baseline Improved Method and Procedure
- B8: Baseline Improved Method and Procedure
- B9: Baseline Improved Method and Procedure
- B10: Baseline Improved Method and Procedure
- B11: Baseline Improved Method and Procedure
- B12: Baseline Improved Method and Procedure
- B13: Baseline Improved Method and Procedure
- B14: Baseline Improved Method and Procedure
- B15: Baseline Improved Method and Procedure
- B16: Baseline Improved Method and Procedure
- B17: Baseline Improved Method and Procedure
- B18: Baseline Improved Method and Procedure
- B19: Baseline Improved Method and Procedure
- B20: Baseline Improved Method and Procedure
- B21: Baseline Improved Method and Procedure
- B22: Baseline Improved Method and Procedure
- B23: Baseline Improved Method and Procedure
- B24: Baseline Improved Method and Procedure
- B25: Baseline Improved Method and Procedure
The parameter $R_M$ was presented as cases 25′–28′ and 35′–37′. In these revised cases, a more theoretically correct equation is used for load ratios $B_2$. Based on these results, the IAM provides results consistent with the DM for the range proposed ($B_2 \leq 2.0$). Unconservatism for the cases examined.

Use of the IAM with $PA$ = 2.0, there is the potential for underestimation of demands if the IAM is used with an approximate second-order analysis.

Notes for Table B1:
1. Cantilever height. Darker values are larger.
2. Horizontal load selected to produce demand-to-capacity ratio of 1.0 for the IAM. Darker values are larger within each group.
4. Axial force in the stability column. Darker values are larger within each group.
5. The $\psi$ parameter for the stability column determined using Equation 3. Values less than 1.0 are highlighted.
6. The ratio for the stability column, with the Euler buckling load $P_{\psi}$ determined in the plane of bending per AISC Specification Equation A-8-5. Darker values are larger.
7. Axial force in the leaning column. Darker values are larger within each group.
8. The sum of the axial forces on the stability column and on the leaning column ($P_{\text{story}} = P_A + P_B$). Darker values are smaller.
9. $R_M = 1 - 0.15P_A/P_{\text{story}}$. AISC Specification Equation A-8-8. Darker values are smaller.
10. The $B_2$ factor determined using Equation 12. Darker values are larger.
11. The $B_3$ factor determined using Equation 13. Darker values are larger.
12. Product of $B_2$ and $B_3$. Darker values of $B_2$ are larger.
17. First-order displacement, $\Delta_1$, determined using Equation 33.
18. Second-order displacement, $\Delta_2$, determined using Equation 34.
20. Ratio of second-order to first-order displacements (Equation 16).
21. The $B'_1$ factor determined using Equation 17.
22. Product of $B'_2$ and $B'_3$.
23. First-order overturning moment $M_{\text{OA}}$ determined using Equation 30.
25. Second-order overturning moment for DM $M_{\text{OA}}$ determined using Equation 32.
26. Stability-column interaction check using AISC Specification Equations H1-1a and H1-1b, using $PA$ and $M_{\text{OA}}$. Values are 1.00 because the lateral force $H$ is selected to produce this value.
27. Stability-column interaction check using AISC Specification Equations H1-1a and H1-1b, using $P_A$ and $B'_2 M_{\text{OA}}$.
28. Stability-column interaction check using AISC Specification Equations H1-1a and H1-1b, using $P_A$ and $B'_3 M_{\text{OA}}$.
29. Ratio of amplified first-order moment to DM moment ($B'_2 B'_3 M_{\text{OA}}$).
30. Ratio of amplified second-order moment to DM moment ($B'_2 M_{\text{OA}}$).
31. Ratio of amplified second-order moment to DM moment ($B'_3 M_{\text{OA}}$).

Figure B1 shows demand-to-capacity ratios from selected cases: 21–28, in which the axial force in column B is varied such that there is a range of $B_2$; 39–44, in which the axial forces in column A and column B are varied such that there is range of axial load ratios $P_A/P_{\psi}$ and $B_2 = 2.0$ is maintained; and 29–38, in which the axial force in column A is varied such that there is a range of both $P_A/P_{\psi}$ and $B_2$. The graphs show that for values of $B_2 \leq 2.0$, the IAM can be used to approximate the DM. Above $B_2 = 2.0$, there is the potential for underestimation of demands if the IAM is used with an approximate second-order analysis. Use of the IAM with $B'_3$ determined using the results of an explicit second-order analysis from Equation 17 does not show this unconservatism for the cases examined.

Based on these results, the IAM provides results consistent with the DM for the range proposed ($B_2 \leq 2.0$).

Reduction of Error by Use of More Exact Formulae

At the bottom of Table B1, cases 25–28 and 35–37 (for which the IAM produced unconservative results for $B_2 > 2.0$) are reanalyzed. (These are presented as cases 25′–28′ and 35′–37′.) In these revised cases, a more theoretically correct equation is used for the parameter $R_M$ based on work by LeMessurier (1977):

\[
R_M = 1 - \left[ 1 - \left( \frac{\pi/2}{3} \right)^2 \right] \frac{P_A}{P_{\text{story}}} = \frac{\pi^2/12 P_A + P_B}{P_A + P_B}
\]

The results show the unconservative error eliminated for all but one case (28′), in which the unconservative error is reduced to 1% (compared to 3% for case 28). The majority of the error in these cases can therefore be attributed to the simplified equation for $R_M$ in AISC Specification Appendix 8, rather than to an inherent deficiency of the IAM.
APPENDIX C
EXAMPLE APPLICATION

The following example illustrates the application of the IAM for stability design of a symmetrical eight-story building with moment frames in one direction. Figure C1 shows the plan. The configuration and loads were selected to result in relatively high second-order effects for a building of this height. For simplicity, only two load combinations are considered: one with service-level lateral loads and one with strength-level lateral loads. Two-dimensional analyses of one frame (on line D) are performed. The LRFD method is employed in this example.

The relevant design data are presented in Tables C1 (general design information), C2 (drift evaluation information), and C3 (strength evaluation information). Gravity loads and lateral loads are both given for the entire building, with 50% of each being tributary to the frame being designed. (The tributary gravity load includes that resisted by leaning columns.) Loads directly on the moment-frame columns include a large cladding load. For simplicity, vertical and lateral load values are unrealistically uniform. Factors $B_2$ and $B_3$ are calculated in advance of any design, assuming the frames have exactly the stiffness required to meet the drift limit for the appropriate lateral loads, including consideration of second-order effects.

Table C2 contains the information used for the serviceability evaluation. Wind loads are based on a uniform pressure of 14.33 psf, and a tall parapet is assumed, resulting in identical wind forces at each story. (Seismic loads are not considered.) The serviceability drift limit is $L/400$; this is a limit on the second-order drift. The required stiffness is the first-order stiffness that will result in the desired second-order drift (the drift limit) in the presence of the vertical loads. (See note 3. The derivation of this required stiffness which is too large to include in this example.) This required stiffness is used to compute the first-order drift $\Delta_1$, which is then used to compute $B_2$. Although second-order effects are addressed in these calculations by means of this determination of required stiffness, no design has yet been performed, and only the predesign information is used.

Similarly, Table C3 is constructed using predesign information. Wind loads are based on a uniform pressure of 21.5 lb/ft$^2$. The first-order lateral stiffness of the system is assumed to be that determined in Table C2 to meet the drift limit in the presence of the serviceability vertical loads. First-order drifts are computed using that stiffness and strength-level lateral loads. (These values are upper bounds based on the assumption of minimum permissible stiffness for serviceability requirements.) The strength-level amplifiers $B_2$ are then computed based on the vertical loads considered in the strength evaluation. These $B_2$ values are then used to determine $B_3$ values, assuming $\alpha P_r/P_{ns} \leq 0.5$.  

Fig. B1. Selected results from parametric study.
A frame is designed and analyzed using analysis software to meet the drift limits in Table C2 with the corresponding vertical and lateral loads. Members are selected (and revised) to result in drifts at or just below the drift limit. Second-order effects are considered, with members appropriately meshed to capture $P$-$\delta$ effects. Results are shown in Table C4. Under “Service,” the ratio of the second-order drift under service-level lateral loads is compared to the drift limit; these ratios are reasonably close to 1.0 for most of the stories, indicating a design optimized for drift control. The design is shown in Figure C2. Column sizes shown are repeated for all moment-frame columns.

The frame is then analyzed using the loads from Table C3 for strength and stability; lateral loads are amplified with the maximum value of $B_3$. Results of the strength and stability evaluation are shown in Table C4. These include the beam and column demand-to-capacity ratios at each story determined using analysis software. As the demand-to-capacity ratios are below 1.0, no redesign is required; member selection to meet drift limits governs the design, and the strength and stability evaluation confirms the acceptability of the design.

The frame is reanalyzed using the DM, and beam and column demand-to-capacity ratios are computed. Additionally, to better understand the simplifications and conservatism of the process employed in this example, $B_3'$ (determined from the analysis using Equation 16) and $B_3^f$ (using Equation 17) are presented, although these values are not used in the evaluation.

Table C4 presents the results of these analyses and design evaluations. The results show a reasonably close correlation between the IAM and the DM design checks. Typical of many moment frame buildings, the member sizes are selected to meet the drift limits, and the minor differences in the strength-design demand-to-capacity ratios do not affect the economy of the design. The differences in demand-to-capacity ratios are due to two conservative measures. First, the use of the allowable drift in determining the $B_3$ amplifier overestimates the value (although it simplifies the design process); this could be avoided by using the analysis-based factor $B_3^f$ form, Equation 17. Second, the use of the largest value of $B_3$ for all stories overestimates the effect at upper stories. Level-by-level application of $B_3$ (or $B_3'$) could address this, as discussed under “Application.”

![Fig. C1. Typical plan of example building.](image)
### Table C1. Pre-Analysis Design Information (General)

<table>
<thead>
<tr>
<th>Level</th>
<th>$L$ (in.)</th>
<th>Dead Load (kips)</th>
<th>Live Load (kips)</th>
<th>$P_{mf}/P_{story}$</th>
<th>$R_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>180</td>
<td>2000</td>
<td>0</td>
<td>0.275</td>
<td>0.959</td>
</tr>
<tr>
<td>7</td>
<td>180</td>
<td>2000</td>
<td>1600</td>
<td>0.275</td>
<td>0.959</td>
</tr>
<tr>
<td>6</td>
<td>180</td>
<td>2000</td>
<td>1600</td>
<td>0.275</td>
<td>0.959</td>
</tr>
<tr>
<td>5</td>
<td>180</td>
<td>2000</td>
<td>1600</td>
<td>0.275</td>
<td>0.959</td>
</tr>
<tr>
<td>4</td>
<td>180</td>
<td>2000</td>
<td>1600</td>
<td>0.275</td>
<td>0.959</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
<td>2000</td>
<td>1600</td>
<td>0.275</td>
<td>0.959</td>
</tr>
<tr>
<td>2</td>
<td>180</td>
<td>2000</td>
<td>1600</td>
<td>0.275</td>
<td>0.959</td>
</tr>
<tr>
<td>1</td>
<td>180</td>
<td>2000</td>
<td>1600</td>
<td>0.275</td>
<td>0.959</td>
</tr>
</tbody>
</table>

Notes:
1. Given information.
2. $R_M = 1 - 0.15P_{mf}/P_{story}$ using AISC Specification Equation A-8-8.

### Table C2. Pre-Analysis Design Information (Drift Evaluation)

<table>
<thead>
<tr>
<th>Level</th>
<th>$H_{service}$ (kips)</th>
<th>$\Delta_{allowable}$ (in.)</th>
<th>$P_{story}$ (kips)</th>
<th>$K_{required}$ (kip/in.)</th>
<th>$\Delta_1$ (in.)</th>
<th>$B_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>20.0</td>
<td>0.450</td>
<td>2,000</td>
<td>56</td>
<td>0.357</td>
<td>1.26</td>
</tr>
<tr>
<td>7</td>
<td>40.0</td>
<td>0.450</td>
<td>4,400</td>
<td>114</td>
<td>0.350</td>
<td>1.29</td>
</tr>
<tr>
<td>6</td>
<td>60.0</td>
<td>0.450</td>
<td>6,800</td>
<td>173</td>
<td>0.347</td>
<td>1.30</td>
</tr>
<tr>
<td>5</td>
<td>80.0</td>
<td>0.450</td>
<td>9,200</td>
<td>231</td>
<td>0.346</td>
<td>1.30</td>
</tr>
<tr>
<td>4</td>
<td>100.0</td>
<td>0.450</td>
<td>11,600</td>
<td>289</td>
<td>0.345</td>
<td>1.30</td>
</tr>
<tr>
<td>3</td>
<td>120.0</td>
<td>0.450</td>
<td>14,000</td>
<td>348</td>
<td>0.345</td>
<td>1.30</td>
</tr>
<tr>
<td>2</td>
<td>140.0</td>
<td>0.450</td>
<td>16,400</td>
<td>406</td>
<td>0.345</td>
<td>1.31</td>
</tr>
<tr>
<td>1</td>
<td>160.0</td>
<td>0.450</td>
<td>18,800</td>
<td>464</td>
<td>0.344</td>
<td>1.31</td>
</tr>
</tbody>
</table>

Notes:
1. Given information.
2. $P_{story}$ is based on 1.0 times the dead load plus 0.25 times the live load for the service drift check.
3. The required stiffness is based on the lateral load and the drift limit, considering the second-order effect: $K_{required} = \frac{H}{\Delta_{allowable} + P_{story}/R_{M}}$.
4. $\Delta_1 = H/K_{required}$
5. The $B_2$ factor using Equation 12 based on $P_{story}$ for the drift load combination (and on $\Delta_1$ and $H_{service}$).
### Table C3. Pre-Analysis Design Information (Strength Evaluation)

<table>
<thead>
<tr>
<th>Level</th>
<th>H (kips)</th>
<th>F (kips)</th>
<th>$K_{required}$ (kip/in.)</th>
<th>$\Delta_1$ (in.)</th>
<th>$P_{story}$ (kips)</th>
<th>$B_2$</th>
<th>$\theta$</th>
<th>$B_3$</th>
<th>$B_3F$ (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>30</td>
<td>30.0</td>
<td>56</td>
<td>0.535</td>
<td>2,400</td>
<td>1.33</td>
<td>0.24</td>
<td>1.09</td>
<td>33.78</td>
</tr>
<tr>
<td>7</td>
<td>60</td>
<td>30.0</td>
<td>114</td>
<td>0.525</td>
<td>5,600</td>
<td>1.40</td>
<td>0.27</td>
<td>1.11</td>
<td>33.78</td>
</tr>
<tr>
<td>6</td>
<td>90</td>
<td>30.0</td>
<td>173</td>
<td>0.521</td>
<td>8,800</td>
<td>1.42</td>
<td>0.28</td>
<td>1.12</td>
<td>33.78</td>
</tr>
<tr>
<td>5</td>
<td>120</td>
<td>30.0</td>
<td>231</td>
<td>0.519</td>
<td>12,000</td>
<td>1.43</td>
<td>0.29</td>
<td>1.12</td>
<td>33.78</td>
</tr>
<tr>
<td>4</td>
<td>150</td>
<td>30.0</td>
<td>289</td>
<td>0.518</td>
<td>15,200</td>
<td>1.44</td>
<td>0.29</td>
<td>1.12</td>
<td>33.78</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
<td>30.0</td>
<td>348</td>
<td>0.518</td>
<td>18,400</td>
<td>1.44</td>
<td>0.29</td>
<td>1.12</td>
<td>33.78</td>
</tr>
<tr>
<td>2</td>
<td>210</td>
<td>30.0</td>
<td>406</td>
<td>0.517</td>
<td>21,600</td>
<td>1.45</td>
<td>0.30</td>
<td>1.13</td>
<td>33.78</td>
</tr>
<tr>
<td>1</td>
<td>240</td>
<td>30.0</td>
<td>464</td>
<td>0.517</td>
<td>24,800</td>
<td>1.45</td>
<td>0.30</td>
<td>1.13</td>
<td>33.78</td>
</tr>
</tbody>
</table>

Notes:
1. Given information.
2. It is conservatively assumed that the stiffness is exactly equal to that required to meet the drift limit.
3. The first-order displacement corresponding to the assumed stiffness and the applied loads.
4. $P_{story}$ is based on 1.2 times the dead load plus 0.5 times the live load for the strength check.
5. The $B_2$ factor determined using Equation 12 based on $P_{story}$ for the strength load combination.
6. The stability coefficient $\theta$ calculated per Equation 1.
7. The $B_3$ factor determined using Equation 15. The maximum value is highlighted.
8. The amplified forces $B_3F$ to be used in a second-order analysis. The maximum value of $B_3$ is used at all levels so that overturning effects at lower levels are not underestimated.

### Fig. C2. Moment frame elevation.
### Table C4. Analysis Results and Design Checks

<table>
<thead>
<tr>
<th>Level</th>
<th>$\Delta_2/\Delta_{all}$</th>
<th>$\alpha P_{1}/P_{ns}$</th>
<th>$\alpha$</th>
<th>$\Delta_1$ (in.)</th>
<th>$\Delta_2$ (in.)</th>
<th>$B'_{2}$</th>
<th>$B'_{3}$</th>
<th>IAM (DCR)</th>
<th>DM (DCR)</th>
<th>Ratio</th>
<th>IAM (DCR)</th>
<th>DM (DCR)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.61</td>
<td>0.07</td>
<td>1.00</td>
<td>0.39</td>
<td>0.49</td>
<td>1.25</td>
<td>1.07</td>
<td>0.13</td>
<td>0.12</td>
<td>1.06</td>
<td>0.14</td>
<td>0.14</td>
<td>1.04</td>
</tr>
<tr>
<td>7</td>
<td>0.96</td>
<td>0.16</td>
<td>1.00</td>
<td>0.59</td>
<td>0.79</td>
<td>1.35</td>
<td>1.10</td>
<td>0.21</td>
<td>0.20</td>
<td>1.04</td>
<td>0.45</td>
<td>0.45</td>
<td>1.01</td>
</tr>
<tr>
<td>6</td>
<td>0.89</td>
<td>0.15</td>
<td>1.00</td>
<td>0.54</td>
<td>0.74</td>
<td>1.36</td>
<td>1.10</td>
<td>0.31</td>
<td>0.30</td>
<td>1.02</td>
<td>0.25</td>
<td>0.25</td>
<td>1.02</td>
</tr>
<tr>
<td>5</td>
<td>0.95</td>
<td>0.21</td>
<td>1.00</td>
<td>0.57</td>
<td>0.79</td>
<td>1.38</td>
<td>1.11</td>
<td>0.27</td>
<td>0.27</td>
<td>1.02</td>
<td>0.45</td>
<td>0.45</td>
<td>1.01</td>
</tr>
<tr>
<td>4</td>
<td>0.93</td>
<td>0.22</td>
<td>1.00</td>
<td>0.56</td>
<td>0.78</td>
<td>1.39</td>
<td>1.11</td>
<td>0.32</td>
<td>0.31</td>
<td>1.02</td>
<td>0.47</td>
<td>0.47</td>
<td>1.01</td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>0.28</td>
<td>1.00</td>
<td>0.60</td>
<td>0.84</td>
<td>1.41</td>
<td>1.11</td>
<td>0.29</td>
<td>0.29</td>
<td>1.01</td>
<td>0.58</td>
<td>0.58</td>
<td>1.01</td>
</tr>
<tr>
<td>2</td>
<td>0.93</td>
<td>0.25</td>
<td>1.00</td>
<td>0.56</td>
<td>0.78</td>
<td>1.38</td>
<td>1.11</td>
<td>0.33</td>
<td>0.32</td>
<td>1.01</td>
<td>0.50</td>
<td>0.50</td>
<td>1.01</td>
</tr>
<tr>
<td>1</td>
<td>0.72</td>
<td>0.29</td>
<td>1.00</td>
<td>0.45</td>
<td>0.59</td>
<td>1.32</td>
<td>1.09</td>
<td>0.33</td>
<td>0.32</td>
<td>1.03</td>
<td>0.65</td>
<td>0.64</td>
<td>1.02</td>
</tr>
</tbody>
</table>

**Notes.**

1. Second-order serviceability drift divided by drift limit.
2. Gridline 5 column $\alpha P_{1}/P_{ns}$ ratio.
3. Gridline 5 column $\alpha$ parameter.
4. $\Delta_1$ from a first-order analysis using strength-level loads.
5. $\Delta_2$ from a second-order analysis using strength-level loads.
6. The $B'_{2}$ factor determined using Equation 16 based on $\Delta_1$ and $\Delta_2$ from the analysis.
7. The $B'_{3}$ factor determined using Equation 17 based on $B'_{2}$.
8. Beam demand-to-capacity ratio from IAM second-order analysis amplified by $B_3$ using AISC Specification Equations H1-1a and H1-1b axial-flexure interaction equations.
10. Ratio of IAM to DM beam demand-to-capacity ratio.
11. Gridline 5 column demand-to-capacity ratio from IAM second-order analysis amplified by $B_3$ using AISC Specification Equations H1-1a and H1-1b axial-flexure interaction equations.
12. Gridline 5 column demand-to-capacity ratio from DM second-order analysis with reduced stiffness using AISC Specification Equations H1-1a and H1-1b axial-flexure interaction equations. $H$ is used for the lateral load.
13. Ratio of IAM to DM gridline 5 column demand-to-capacity ratio.
Steel Structures Research Update

Self-Centering Column Base Connections with Friction Dampers

JUDY LIU

INTRODUCTION

R
ecent work on a self-centering column base connection with friction dampers is highlighted. This research is a collaborative effort by Senior Researcher Massimo Latour and Professor Gianvittorio Rizzano from the University of Salerno, Italy, and Professors Aldina Santiago and Luis Simões da Silva of the University of Coimbra, Portugal. The work builds on the researchers’ combined expertise on “free from damage” beam-to-column connections, friction materials for supplemental damping, and experimental testing and analytical modeling of a variety of connections.

Self-centering and rocking column bases have captured the interest of researchers across the globe. Some of the self-centering column base research in the past decade has originated from Japan, China, the United Kingdom, Taiwan, Canada, and the United States. Hayashi et al. (2018) developed a self-centering, rocking composite frame with post-tensioned (PT), concrete-filled tube (CFT) columns combined with a moment-resisting frame (MRF) with low-yield (LY) fuses. Kamperidis et al. (2018) proposed a partial-strength, self-centering steel column base with post-tensioning and replaceable, hourglass steel yielding devices. Freddi et al. (2017) and Chen et al. (2014) investigated rocking, PT column bases with friction devices to dissipate energy. PT column bases with yielding, buckling restrained steel (BRS) plates have been developed for CFT and wide-flange columns in MRFs (Wang et al., 2019; Chi and Liu, 2012, respectively). The PT column bases studied by Chi and Liu (2012) were part of a larger effort; Sause et al. (2010) conducted extensive design and validation of self-centering moment resisting frames and concentrically braced frames. Wiebe et al. (2013) investigated controlled rocking steel frames with configurations that implement rocking at upper sections to accommodate higher modes. Rocking steel-braced frames using post-tensioning and replaceable seismic fuses were developed by Eatherton et al. (2014).

The Salerno-Coimbra team has developed an alternative self-centering column base solution to minimize initial costs and economic losses. Specific objectives are to limit damage and residual drifts, with connection components that are easy to repair or replace if needed. The self-centering column base has been validated through quasi-static cyclic testing and pseudo-dynamic testing. The self-centering connection has also been investigated through numerical time-history analyses of moment-resisting frames comparing conventional, fixed column bases to self-centering column bases. Some highlights of the research are presented.

BACKGROUND AND MOTIVATION

The research is focused on design of steel moment-resisting frames (MRFs) to achieve seismic performance objectives while minimizing economic losses. The traditional design strategy is to develop plastic hinges in the beam ends and at the column bases with strong columns and full-strength connections. This strategy results in significant damage and residual drift due to the permanent, plastic deformations. Given the associated costs and questions related to repa rability of the structure, the researchers sought alternative energy dissipation methods and connection designs.

With the goal of minimizing damage, the researchers explored partial strength connections with friction dampers. These types of connections were initially proposed by Grigorian et al. (1993). This work was followed by numerous other theoretical, experimental, and numerical studies (Latour et al., 2019). Specifically, in New Zealand, researchers developed the sliding hinge joint (SHJ) (e.g., Khoo et al., 2012; Ramhormozian et al., 2014) for a beam to column connection. The SHJ has a friction connection at the bottom beam flange, with friction pads made of mild steel, aluminum, brass, or abrasion-resistant steel. A European adaptation of this connection uses a bolted T-stub at the top flange and a shop-assembled friction damping device bolted to the bottom flange (Latour et al., 2018). As shown in Figure 1, the friction damping device consists of a slotted haunch; friction pads; L-stubs to connect the haunch to the column; and pretensioned bolts clamping the haunch, friction pads, and L-stubs. Energy dissipation is achieved through slip at the friction pads as the beam rotates about the stem of the T-stub. The same principles can be further adapted for use at a column base.
The success with the adapted sliding hinge connection is tempered by residual drift issues. Although the friction connection does not experience the same damage as a plastic hinge, the connection experiences similar permanent deformations due to its high unloading stiffness. “Indeed, although these connections are very effective from the point of view of the damage avoidance, they still provide significant problems related to the low self-centering capacity.” (Latour et al., 2019) To address this, the researchers also explored a supplemental self-centering solution for the column base.

As briefly described earlier, a number of researchers have proposed self-centering and rocking column base solutions. The Salerno–Coimbra team evaluated the various solutions in the development of their own column base. The team looked first to low-damage friction connections developed and tested by Borzouie et al. (2015) and shown to avoid problems with axial shortening of columns due to yielding and local buckling (MacRae et al., 2009). Then, for self-centering solutions, the researchers acknowledged the benefits of using long PT bars extending into the basement level to avoid yielding of those PT bars (e.g., Chi and Liu, 2012), but they suspected difficulties in repair and replacement. The researchers also noted a related objective to avoid any connection of the PT bars to a concrete foundation. Meanwhile, the researchers found promise in a study utilizing a tension-limiting base level hinge. The base hinge consisted of prestressed Ringfeder springs and vertical friction plates. The Ringfeder springs were prestressed by a tightened bolt through their center, and the friction plates did not engage until the gravity load and Ringfeder spring prestress was exceeded (Gledhill et al., 2008). These and additional studies are described in more detail in Latour et al. (2019). From those studies, the researchers proposed to “keep the layout of the connection as simple as possible providing, other than the self-centering capacity, additional benefits such as the absence of interaction with the concrete foundation and the limited size of the connection which is, overall, similar or lower than the size of the cover plates employed to realize a traditional column splice connection” (Latour et al., 2019).

THE PROPOSED COLUMN BASE CONFIGURATION

For their self-centering column base, the researchers propose a column-splice with friction pads and threaded bars with Belleville disk springs, located just above a traditional full-strength base plate connection, as shown in Figure 2(a). The benefits of the proposed connection follow the researchers’ objectives of providing a simple, self-centering connection detail that is not larger than a conventional column splice connection and does not have any attachment to the concrete foundation. As such, damage and residual drifts are limited, and the connection components are expected to be easy to repair or replace if needed.

In the proposed self-centering column base, the moment-rotation response is at the column splice and is governed by the component behavior. As shown in Figure 2(a), there is a stiffened base plate connection and a column stub. The column stub is spliced to the rest of the column with flange and web plates and friction pads with pretensioned bolts. Slotted and oversized holes in the column flanges and web above the splice are used to accommodate the rotation—that is, gap opening—at the column flanges. Threaded bars on each side of the column web are anchored to stiffener plates above and below the column splice, with a system of Belleville disk springs between the nuts and stiffener plates. The contributions of the friction pads, threaded bars, and Belleville disk
springs to the moment-rotation behavior can be idealized in a mechanical model of equivalent springs, as shown in Figure 2(b). Springs $F_w$ and $F_f$ represent the friction pads on the column web and flanges. The $F_w$ and $F_f$ springs have infinite stiffness up until the slip force and zero stiffness after slip. The translational spring $F_{tb}$ models the axial behavior of the threaded bars, which work in series with the system of disk springs, $F_{ds}$. The force deformation behavior of the $F_{tb}$ and $F_{ds}$ springs are dependent on the number and properties of the threaded bars and the number and properties of the disk springs working in series and in parallel.

As shown in Figure 3, the disk springs can be stacked to work in parallel or in series, and in this manner, the system of disk springs can be tuned to the desired stiffness. Using the equations developed by Latour et al. (2019) for the equivalent springs, the typical moment-rotation behavior of the connection is shown to follow a flag shape hysteresis, as shown in Figure 2(c). At the bending moment $M_0$, the initial axial load in the column and the prestress of the threaded bars have been offset, and the bending moment $M_1$ is the contribution to the bending moment due to the friction pads. At the top of loading branch 1, the moment $M_2$ represents the decompression moment corresponding to slip in the friction pads and gap opening (i.e., rotation) at the column splice. Loading branch 2 corresponds to slip in the friction pads and loading governed by the stiffnesses of the threaded bars, disk springs, and column in bending. Unloading branches 3 and 4 are governed by the same behavior as the loading branches 1 and 2. The connection returns to zero moment and zero rotation—that is, no residual plastic deformation.

![Fig. 2. (a) Connection assembly; (b) mechanical model; (c) theoretical moment-rotation relationship.](image)

![Fig. 3. Belleville disk spring stacking methods.](image)
EXPERIMENTAL INVESTIGATION

The proposed self-centering column base connection has been validated through quasi-static cyclic testing and pseudo-dynamic testing. A cantilever specimen representing the bottom half of a first-story column was designed using the principles outlined in Latour et al. (2019). The column was a 9.44-in.-deep, 9.44-in.-wide H-section (HEB240) with a yield stress of 40 ksi (S275 steel). The connection plates were also S275 steel. The friction pads were S275 steel plates with a 0.012-in. thermal spray aluminum coating. Prior to the cyclic and pseudo-dynamic testing, the friction pads were tested and characterized by Silva (2016).

The test specimens were subjected to axial and lateral loads. As shown in Figure 4, an actuator placed on top of the specimen was used to apply the axial load under load control. A displacement-controlled actuator applied increasing, cyclic lateral displacements at the top of the specimen. Given the limitations of the test equipment, the axial load was limited to 25% of the squash load; from the applied lateral load, the bending moment at the column splice was limited to 95% of the plastic bending moment of the column. Figure 5 shows the test frame, the connection being assembled, and the complete column base before the test.

Cyclic Testing

Four cyclic tests with and without the threaded bars, and with different axial loads, were conducted. Axial loads of 25% or 12.5% of the squash load were applied and held constant for the duration of the test. The test specimens therefore represented an internal moment frame column that does not experience large changes in axial force during an earthquake. For the lower applied axial force, the axial force in the threaded bars, \( F_{tb} \), was increased but could not be increased to the level needed to ensure recentering of the column (Latour et al., 2019).

The cyclic tests validated the design of the self-centering column base connection, highlighting the importance of the recentering bars and the total axial load in the column. Figure 6 compares moment-rotation response—with and without the threaded, recentering bars—for an applied axial load of 25% and 12.5% of the squash load. Both tests with the 25% axial load ratio exhibited self-centering behavior, with residual rotations of 2.1 mrad and 4.1 mrad with and without the recentering bars, respectively. These rotations were both lower than common construction tolerances on the order of 5 mrad (Latour et al., 2019). Figure 6(a) qualitatively shows the improvement in self-centering behavior.

Fig. 4. Front and side views of the test setup.
with the bars. The test specimens with 12.5% axial load ratio exhibited much higher residual rotations. The recentering bars reduced the residual rotations from almost 50 mrad to 31 mrad. However, higher-capacity bars were needed to provide recentering behavior. Additional details for the cyclic tests can be found in Latour et al. (2019).

**Pseudo-Dynamic Testing**

The proposed self-centering column base connection was further validated with pseudo-dynamic tests. For a more realistic dynamic-response history, a computer simulation was used to account for damping and inertial effects, and the physical test provided restoring force and displacement data for the structure. The structure was idealized as a discrete-parameter system with one degree of freedom. With this idealization, the pseudo-dynamic tests could be conducted with the same test setup used for the cyclic testing. Two ground motions, Kobe (Japan, 1995) and Spitak (Armenia, 1988), were selected to compare results for ground motions with different characteristics. “Kobe is a seismic event inducing a high number of large amplitude cycles. Spitak is characterized mainly by two large reversal and many low amplitude cycles. The scale factor of the seismic events was selected in order to achieve in the connection, approximately, a rotation of 40 mrad” (Latour et al., 2019). An axial load ratio of 25% was used for all tests. Specimens with and without the recentering bars were tested for the Kobe ground motion. The results for the Kobe tests [scale factor 1.4 (PGA = 0.35g)] will be briefly presented. Results for the Spitak test can be found in Latour et al. (2019).

The pseudo-dynamic test results highlighted the role of the recentering bars. Figure 7 shows the moment-rotation responses for the Kobe ground motion and the improvement in the self-centering behavior of the column base with the recentering bars. The bars reportedly reduced the residual rotation from 5.2 mrad to 1.7 mrad. The improved self-centering behavior can also be seen in the reduction of residual drift in the displacement time history plots in Figure 8.

**NUMERICAL SIMULATIONS**

The proposed self-centering column base connection was further investigated through numerical time-history analyses of moment-resisting frames. Two four-bay, six-story MRFs were designed, one with conventional, fixed column bases and one with self-centering connections. The MRFs had 20-ft bays and 10.5-ft story heights with the exception of the 11.5-ft first story. Preliminary beam sizing was based on dead and live loads of 84 psf and 42 psf. The MRF members were designed for a region of high seismicity (PGA = 0.35g) and dense sand, gravel, or stiff clay.

The self-centering column base connections were represented with a mechanical model, an assembly of springs and gap elements. The mechanical model was able to capture the moment-rotation responses from the experimental tests. Four bilinear springs in parallel with gap elements simulated the hysteretic response of the friction dampers.
Fig. 6. Moment-rotation responses.
Fig. 7. Moment-rotation responses for the Kobe ground motion.

Fig. 8. Displacement time histories for Kobe ground motion.
A central bilinear spring simulated the initial prestress and the hysteretic response of the recentering bars. Properties of the springs were calibrated to the experimental data. Meanwhile, the bolted beam-to-column connections with friction dampers (Figure 1) were modeled according to Latour et al. (2018). A damping ratio of 5% was used.

The time-history analyses were conducted for a simulated seismic event. That simulated event was based on eight natural ground motions. Figure 9 compares the roof displacement time histories for the conventional and self-centering column base MRFs. For the conventional full-strength column base connections, the residual roof displacement at the top of the building was 13.8 in., corresponding to 18 mrad of drift. Use of self-centering column base connections reduced the residual displacement by 85%, to 2.4 in., or 3 mrad drift, well within acceptable levels.

**SUMMARY AND FUTURE WORK**

The Salerno-Coimbra team has proposed a self-centering column base connection designed to minimize initial costs and economic losses. The proposed connection is a column-splice with friction pads and threaded bars with Belleville disk springs, located just above a traditional full-strength base plate connection. With the self-centering column base connection, damage and residual drifts are limited, and the connection components are expected to be easy to repair or replace if needed. The self-centering column base has been validated through quasi-static cyclic testing and quasi-dynamic testing. The self-centering connection was further investigated through numerical time-history analyses of moment resisting frame (MRFs) comparing conventional, fixed column bases to self-centering column bases. The experimental and numerical results are promising. The research team looks to extend the work to other configurations to more broadly validate their self-centering column base concept.

**ACKNOWLEDGMENTS**

Special thanks to Professor Aldina Santiago for all of the materials, coordination, edits, and other feedback and contributions. The work of the entire research team, including graduate research assistants, is appreciated.

The researchers would like to gratefully acknowledge the sponsorship from the European Commission within RFCS Research & Innovation by research grant RFSC-CT-2015-00022. Any findings, recommendations, or other material within are those of the researchers and do not necessarily reflect the views of the sponsors.

**REFERENCES**


Guide for Authors

Scope  *Engineering Journal* is dedicated to the improvement and advancement of steel construction. Its pages are open to all who wish to report on new developments or techniques in steel design, research, the design and/or construction of new projects, steel fabrication methods, or new products of significance to the uses of steel in construction. Only original papers should be submitted.

General  Papers intended for publication should be submitted by email Margaret Matthew, editor, at matthew@aisc.org.

The articles published in the *Engineering Journal* undergo peer review before publication for (1) originality of contribution; (2) technical value to the steel construction community; (3) proper credit to others working in the same area; (4) prior publication of the material; and (5) justification of the conclusion based on the report.

All papers within the scope outlined above will be reviewed by engineers selected from among AISC, industry, design firms, and universities. The standard review process includes outside review by an average of three reviewers, who are experts in their respective technical area, and volunteers in the program. Papers not accepted will not be returned to the author. Published papers become the property of the American Institute of Steel Construction and are protected by appropriate copyrights. No proofs will be sent to authors. Each author receives three copies of the issue in which his contribution appears.

Manuscripts  Manuscripts must be provided in Microsoft Word format. Include a PDF with your submittal so we may verify fonts, equations and figures. View our complete author guidelines at aisc.org/ej.